Chapter 1

MEASUREMENTS

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand what is Physics.
2. Understand that all physical quantities consist of a numerical magnitude and a unit.
3. Recall the following base quantities and their units; mass (kg), length (m), time (s), current (A), temperature (K), luminous intensity (cd) and amount of substance (mol).
4. Describe and use base units, supplementary units, and derived units.
5. Understand and use the scientific notation.
6. Use the standard prefixes and their symbols to indicate decimal sub-multiples or multiples to both base and derived units.
7. Understand and use the conventions for indicating units.
8. Understand the distinction between systematic errors and random errors.
9. Understand and use the significant figures.
10. Understand the distinction between precision and accuracy.
11. Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties.
12. Quote answers with correct scientific notation, number of significant figures and units in all numerical and practical work.
13. Use dimensionality to check the homogeneity of physical equations.
14. Derive formulae in simple cases using dimensions.

Eversince man has started to observe, think and reason he has been wondering about the world around him. He tried to find ways to organize the disorder prevailing in the observed facts about the natural phenomena and material things in an orderly manner. His attempts resulted in the birth of a single discipline of science, called natural philosophy. There was a huge increase in the volume of scientific knowledge up till the beginning of nineteenth century and it was found necessary to classify the study of nature into two branches, the biological sciences which deal with living things and physical sciences which concern with non-living things. Physics is an important and basic part of physical sciences besides its other disciplines such as chemistry, astronomy, geology etc. Physics is an experimental science and the scientific method emphasizes the need of accurate measurement of various measurable features of different phenomena or of man made objects. This chapter emphasizes the need of thorough understanding and practice of measuring techniques and recording skills.

Areas of Physics

Mechanics
Heat & thermodynamics
Electromagnetism
Optics
Sound
Hydrodynamics
Special relativity
General relativity
Quantum mechanics
Atomic physics
Molecular physics
Nuclear physics
Solid-state physics
Particle physics
Superconductivity
Super fluidity
Plasma physics
Magneto hydrodynamics
Space physics

1.1 INTRODUCTION TO PHYSICS

At the present time, there are three main frontiers of fundamental science. First, the world of the extremely large, the universe itself, Radio telescopes now gather information from the far side of the universe and have recently detected, as radio waves, the "firelight" of the big bang which probably started off the expanding universe nearly 20 billion years ago. Second, the world of the extremely small, that of the particles such as, electrons, protons, neutrons, mesons and others. The third frontier is the world of complex matter. It is also the World of "middle-sized" things, from molecules at one extreme to the Earth at the other. This is all fundamental physics, which is the heart of science.

But what is physics? According to one definition, physics deals with the study of matter and energy and the relationship between them. The study of physics involves investigating such things as the laws of motion, the structure of space and time, the nature and type of forces that hold different materials together, the interaction between different particles, the interaction of electromagnetic radiation with matter and so on.

Interdisciplinary areas of Physics

Astrophysics
Biophysics
Chemical physics
Engineering physics
Geophysics
Medical physics
Physical oceanography
Physics of music

By the end of 19^{th} century many physicists started believing that every thing about physics has been discovered. However, about the beginning of the twentieth century many new experimental facts revealed that the laws formulated by the previous investigators need modifications. Further researches gave birth to many new disciplines in physics such as nuclear physics which deals with atomic nuclei, The measurement of a base quantity involves two steps: first, the choice of a standard, and second, the establishment of a procedure for comparing the quantity to be measured with the standard so that a number and a unit are determined as the measure of that quantity.

An ideal standard has two principal characteristics: it is accessible and it is invariable. These two requirements are often incompatible and a compromise has to be made between them.

1.3 INTERNATIONAL SYSTEM OF UNITS

In 1960, an international committee agreed on a set of definitions and standard to describe the physical quantities. The system that was established is called the System International (SI).

Due to the simplicity and convenience with which the units in this system are amenable to arithmetical manipulation, it is in universal use by the world's scientific community and by most nations. The system international (SI) is built up from three kinds of units: base units, supplementary units and derived units.

Table 1.1

Physical Quantity	SI Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Intensity of light	candela	cd
Amount of substance	mole	mol

Base Units

There are seven base units for various physical quantities namely: length, mass, time, temperature, electric current, luminous intensity and amount of a substance (with special reference to the number of particles).

The names of base units for these physical quantities together with symbols are listed in Table 1.1. Their standard definitions are given in the Appendix 1.

Supplementary Units

The General Conference on Weights and Measures has not yet classified certain units of the SI under either base units or derived units. These SI units are called supplementary units. For the time being this class contains only two units of purely geometrical quantities, which are plane angle and the solid angle (Table1.2).

Table 1.2
Supplementary units

Physical Quantity	SI Unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

particle physics which is concerned with the ultimate particles of which the matter is composed, relativistic mechanics which deals with velocities approaching that of light and solid state physics which is concerned with the structure and properties of solids, but this list is by no means exhaustive.

Physics is most fundamental of all sciences and provides other branches of science, basic principles and fundamental laws. This overlapping of physics and other fields gave birth to new branches such as physical chemistry, biophysics, astrophysics, health physics etc. Physics also plays an important role in the development of technology and engineering.

Science and technology are a potent force for change in the outlook of mankind. The information media and fast means of communications have brought all parts of the world in close contact with one another. Events in one part of the world immediately reverberate round the globe.

We are living in the age of information technology. The computer networks are products of chips developed from the basic ideas of physics. The chips are made of silicon. Silicon can be obtained from sand. It is upto us whether we make a sandcastle or a computer out of it.

Do You Know?

Computer chips are made from wafers of the metalloid silicon, a semiconductor.

For Your Information

Order of magnitude of some distances

1.2 PHYSICAL QUANTITIES

The foundation of physics rests upon physical quantities in terms of which the laws of physics are expressed. Therefore, these quantities have to be measured accurately. Among these are mass, length, time, velocity, force, density, temperature, electric current, and numerous others.

Physical quantities are often divided into two categories: base quantities and derived quantities. Derived quantities are those whose definitions are based on other physical quantities. Velocity, acceleration and force etc. are usually viewed as derived quantities. Base quantities are not defined in terms of other physical quantities. The base quantities are the minimum number of those physical quantities in terms of which other physical quantities can be defined. Typical examples of base quantities are length, mass and time.

Radian

The radian is the plane angle between two radii of a circle which cut off on the circumference an arc, equal in length to the radius, as shown in Fig. 1.1 (a).

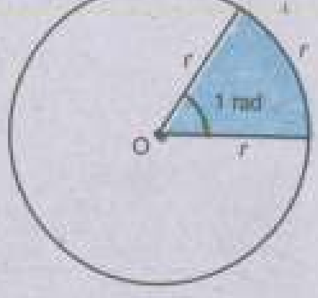


Fig. 1.1(a)

Steradian

The steradian is the solid angle (three-dimensional angle) subtended at the centre of a sphere by an area of its surface equal to the square of radius of the sphere. (Fig. 1.1 b).

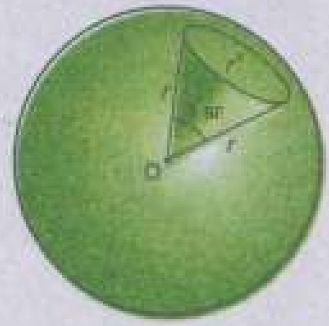


Fig. 1.1(b)

Derived Units

SI units for measuring all other physical quantities are derived from the base and supplementary units. Some of the derived units are given in Table. 1.3.

Table 1.3

Physical quantity	Unit	Symbol	In terms of base units
Force	newton	N	$kg\ m\ s^{-2}$
Work	joule	J	$N\ m = kg\ m^2\ s^{-2}$
Power	watt	W	$J\ s^{-1} = kg\ m^2\ s^{-3}$
Pressure	pascal	Pa	$N\ m^{-2} = kg\ m^{-1}\ s^{-2}$
Electric charge	coulomb	C	A s

Scientific Notation

Numbers are expressed in standard form called scientific notation, which employs powers of ten. The internationally accepted practice is that there should be only one non-zero digit left of decimal. Thus, the number 134.7 should be written as 1.347×10^2 and 0.0023 should be expressed as 2.3×10^{-3}.

Conventions for Indicating Units

Use of SI units requires special care, more particularly in writing prefixes.

Following points should be kept in mind while using units.

(i) Full name of the unit does not begin with a capital letter even if named after a scientist e.g.,newton.

(ii) The symbol of unit named after a scientist has initial capital letter such as N for newton.

(iii) The prefix should be written before the unit without any space, such as 1×10^{-3} m is written as 1 mm. Standard prefixes are given in table 1.4.

(iv) A combination of base units is written each with one space apart. For example, newton metre is written as N m.

(v) Compound prefixes are not allowed. For example, 1μμF may be written as 1pF.

(vi) A number such as 5.0×10^4 cm may be expressed in scientific notation as 5.0×10^2 m.

(vii) When a multiple of a base unit is raised to a power, the power applies to the whole multiple and not the base unit alone. Thus, $1\ km^2 = 1\ (km)^2 = 1 \times 10^6\ m^2$.

(viii) Measurement in practical work should be recorded immediately in the most convenient unit, e.g., micrometer screw gauge measurement in mm, and the mass of calorimeter in grams (g). But before calculation for the result, all measurements must be converted to the appropriate SI base units.

Table 1.4
Some Prefixes for Powers of Ten

Factor	Prefix	Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{1}	deca	da
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

1.4 ERRORS AND UNCERTAINTIES

All physical measurements are uncertain or imprecise to some extent. It is very difficult to eliminate all possible errors or uncertainties in a measurement. The error may occur due to (1) negligence or inexperience of a person (2) the faulty apparatus (3) inappropriate method or technique. The uncertainty may occur due to inadequacy or limitation of an instrument, natural variations of the object being measured or natural imperfections of a person's senses. However, the uncertainty is also usually described as an error in a measurement. There are two major types of errors.

(i) **Random error** (ii) **Systematic error**

Random error is said to occur when repeated measurements of the quantity, give different values under the same conditions. It is due to some unknown causes. Repeating the measurement several times and taking an average can reduce the effect of random errors.

Systematic error refers to an effect that influences all measurements of a particular quantity equally. It produces a consistent difference in readings. It occurs to some definite rule. It may occur due to zero error of instruments, poor calibration of instruments or incorrect markings etc. Systematic error can be reduced by comparing the instruments with another which is known to be more accurate. Thus for systematic error, a correction factor can be applied.

1.5 SIGNIFICANT FIGURES

As stated earlier physics is based on measurements. But unfortunately whenever a physical quantity is measured, there is inevitably some uncertainty about its determined value. This uncertainty may be due to a number of reasons. One reason is the type of instrument, being used. We know that every measuring instrument is calibrated to a certain smallest division and this fact put a limit to the degree of accuracy which may be achieved while measuring with it. Suppose that we want to measure the length of a straight line with the help of a metre rod calibrated in millimetres. Let the end point of the line lies between 10.3 and 10.4 cm marks. By convention, if the end of the line does not touch or cross the midpoint of the smallest division, the reading is confined to the previous division. In case the end of the line seems to be touching or have crossed the midpoint, the reading is extended to the next division.

For Your Information

	Interval (s)
Age of the universe	5×10^{17}
Age of the Earth	1.4×10^{17}
One year	3.2×10^{7}
One day	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period of audible sound waves	1×10^{-3}
Period of typical radio waves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}

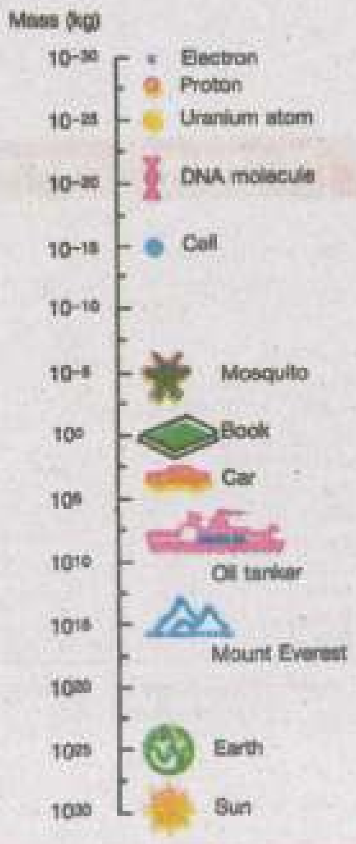
Approximate Values of Some Time Intervals

By applying the above rule the position of the edge of a line recorded as 12.7 cm with the help of a metre rod calibrated in millimetres may lie between 12.65 cm and 12.75 cm. Thus in this example the maximum uncertainty is ± 0.05 cm. It is, in fact, equivalent to an uncertainty of 0.1 cm equal to the least count of the instrument divided into two parts, half above and half below the recorded reading.

The uncertainty or accuracy in the value of a measured quantity can be indicated conveniently by using significant figures. The recorded value of the length of the straight line i.e. 12.7 cm contains three digits (1, 2, 7) out of which two digits (1 and 2) are accurately known while the third digit i.e. 7 is a doubtful one. As a rule:

In any measurement, the accurately known digits and the first doubtful digit are called significant figures.

Interesting Information



Order of magnitude of some masses.

In other words, a significant figure is the one which is known to be reasonably reliable. If the above mentioned measurement is taken by a better measuring instrument which is exact upto a hundredth of a centimetre, it would have been recorded as 12.70 cm. In this case, the number of significant figures is four. Thus, we can say that as we improve the quality of our measuring instrument and techniques, we extend the measured result to more and more significant figures and correspondingly improve the experimental accuracy of the result. While calculating a result from the measurements, it is important to give due attention to significant figures and we must know the following rules in deciding how many significant figures are to be retained in the final result.

(i) All digits 1,2,3,4,5,6,7,8,9 are significant. However, zeros may or may not be significant. In case of zeros, the following rules may be adopted.

a) A zero between two significant figures is itself significant.

b) Zeros to the left of significant figures are not significant. For example, none of the zeros in 0.00467 or 02.59 is significant.

c) Zeros to the right of a significant figure may or may not be significant. In decimal fraction, zeros to the right of a significant figure are significant. For example, all the zeros in 3.570 or 7.4000 are significant. However, in integers such as 8,000 kg, the number of significant zeros is determined by the accuracy of the measuring instrument. If the measuring scale has a least count of 1 kg then there are four significant figures written in scientific notation as 8.000×10^3 kg. If the least count of the scale is 10 kg, then the number of significant figures will be 3 written in scientific notation as 8.00×10^3 kg and so on.

d) When a measurement is recorded in scientific notation or standard form, the figures other than the powers of ten are significant figures. For example, a measurement recorded as 8.70×10^4 kg has three significant figures.

(ii). In multiplying or dividing numbers, keep a number of significant figures in the product or quotient not more than that contained in the least accurate factor i.e., the factor containing the least number of significant figures. For example, the computation of the following using a calculator, gives

$$\frac{5.348 \times 10^{-2} \times 3.64 \times 10^{4}}{1.336} = 1.45768982 \times 10^{3}$$

As the factor 3.64×10^4, the least accurate in the above calculation has three significant figures, the answer should be written to three significant figures only. The other figures are insignificant and should be deleted. While deleting the figures, the last significant figure to be retained is rounded off for which the following rules are followed.

a) If the first digit dropped is less than 5, the last digit retained should remain unchanged.

b) If the first digit dropped is more than 5, the digit to be retained is increased by one.

c) If the digit to be dropped is 5, the previous digit which is to be retained is increased by one if it is odd and retained as such if it is even. For example, the following numbers are rounded off to three significant figures as follows. The digits are deleted one by one.

43.75	is rounded off as	43.8
56.8546	is rounded off as	56.8
73.650	is rounded off as	73.6
64.350	is rounded off as	64.4

Do You Know ?

Mass can be thought of as a form of energy. In fact the mass is highly concentrated form of energy. Einstein's famous equation, $E=mc^2$ means

Energy = mass x speed of Light^2

According to this equation 1 kg mass is actually 9×10^{16} J energy.

Following this rule, the correct answer of the computation given in section (ii) is 1.46×10^3.

(iii) In adding or subtracting numbers, the number of decimal places retained in the answer should equal the smallest number of decimal places in any of the quantities being added or subtracted. In this case, the number of significant figures is not important. It is the position of decimal that matters. For example, suppose we wish to add the following quantities expressed in metres.

	i)	72.1	ii)	2.7543
		3.42		4.10
		0.003		1.273
		75.523		8.1273
Correct answer:		75.5 m		8.13 m

In case (i) the number 72.1 has the smallest number of decimal places, thus the answer is rounded off to the same position which is then 75.5 m. In case (ii), the number 4.10 has the smallest number of decimal places and hence, the answer is rounded off to the same decimal positions which is then 8.13 m.

For your information

We use many devices to measure physical quantities, such as length, time, and temperature. They all have some limit of precision.

1.6 PRECISION AND ACCURACY

In measurements made in physics, the terms precision and accuracy are frequently used. They should be distinguished clearly. The precision of a measurement is determined by the instrument or device being used and the accuracy of a measurement depends on the fractional or percentage uncertainty in that measurement.

For example, when the length of an object is recorded as 25.5 cm by using a metre rod having smallest division in millimetre, it is the difference of two readings of the initial and final positions. The uncertainty in the single reading as discussed before is taken as ± 0.05 cm which is now doubled and is called absolute uncertainty equal to ±0.1cm. Absolute uncertainty, in fact, is equal to the least count of the measuring instrument.

Precision or absolute uncertainty (least count) = ± 0.1 cm

$$\text{Fractional uncertainty} = \frac{0.1\,\text{cm}}{25.5\,\text{cm}} = 0.004$$

$$\text{Percentage uncertainty} = \frac{0.1\,\text{cm}}{25.5\,\text{cm}} \times 100 = 0.4\%$$

Another measurement taken by vernier callipers with least count as 0.01 cm is recorded as 0.45 cm. It has

Precision or absolute uncertainty (least count) = ± 0.01 cm

$$\text{Fractional uncertainty} = \frac{0.01\,\text{cm}}{0.45\,\text{cm}} = 0.02$$

$$\text{Percentage uncertainty} = \frac{0.1\,\text{cm}}{0.45\,\text{cm}} \times 100 = 2.0\%$$

Thus the reading 25.5 cm taken by metre rule is although less precise but is more accurate having less percentage uncertainty or error.

Whereas the reading 0.45 cm taken by vernier callipers is more precise but is less accurate. In fact, it is the relative measurement which is important. The smaller a physical quantity, the more precise instrument should be used. Here the measurement 0.45 cm demands that a more precise instrument, such as micrometre screw gauge, with least count 0.001 cm, should have been used. Hence, we can conclude that :

For your information

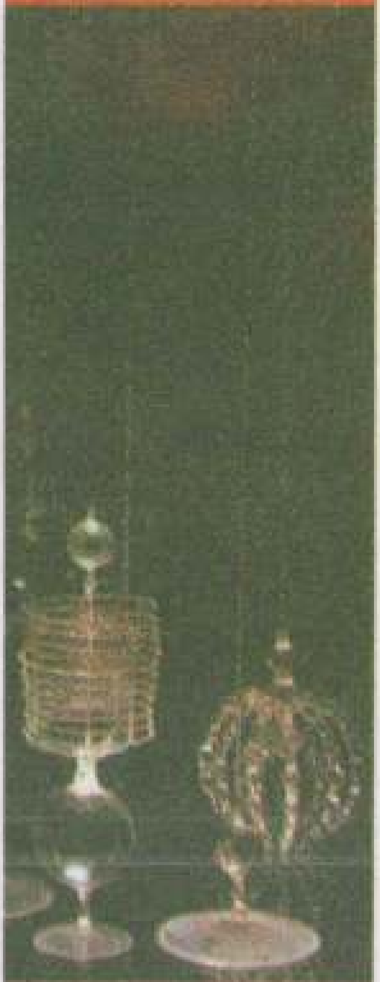
Colour printing uses just four colours- cyan, magenta, yellow and black to produce the entire range of colours. All the colours in this book have been made from just these four colours.

A precise measurement is the one which has less absolute uncertainty and an accurate measurement is the one which has less fractional or percentage uncertainty or error.

1.7 ASSESSMENT OF TOTAL UNCERTAINTY IN THE FINAL RESULT

To assess the total uncertainty or error, it is necessary to evaluate the likely uncertainties in all the factors involved in that calculation. The maximum possible uncertainty or error in the final result can be found as follows. The proofs of these rules are given in Appendix 2.

For your information



These are not decoration pieces of glass but are the earliest known exquisite and sensitive thermometers, built by the Accademia del Cimento (1657-1667), in Florence. They contained alcohol, some times coloured red for easier reading.

1. For addition and subtraction

Absolute uncertainties are added: For example, the distance x determined by the difference between two separate position measurements

$x_1 = 10.5 \pm 0.1$ cm and $x_2 = 26.8 \pm 0.1$ cm is recorded as

$$x = x_2 - x_1 = 16.3 \pm 0.2 \text{ cm}$$

2. For multiplication and division

Percentage uncertainties are added. For example the maximum possible uncertainty in the value of resistance R of a conductor determined from the measurements of potential difference V and resulting current flow I by using $R = V/I$ is found as follows:

$$V = 5.2 \pm 0.1 \text{ V}$$

$$I = 0.84 \pm 0.05\text{A}$$

The %age uncertainty for V is $= \frac{0.1\text{ V}}{5.2\text{ V}} \times 100$ = about 2%

The %age uncertainty for I is $= \frac{0.05\text{ A}}{0.84\text{ A}} \times 100$ = about 6%

Hence total uncertainty in the value of resistance R when V is divided by I is 8%. The result is thus quoted as

$R = \frac{5.2\text{ V}}{0.84\text{ A}} = 6.19\text{ VA}^{-1}$ = 6.19 ohms with a % age uncertainty of 8%

that is $R = 6.2 \pm 0.5$ ohms

The result is rounded off to two significant digits because both V and R have two significant figures and uncertainty, being an estimate only, is recorded by one significant figure.

3. For power factor

Multiply the percentage uncertainty by that power. For example, in the calculation of the volume of a sphere using

$$V = \frac{4}{3}\pi r^3$$

%age uncertainty in V = 3 x % age uncertainty in radius r.

As uncertainty is multiplied by power factor, it increases the precision demand of measurement. If the radius of a small sphere is measured as 2.25 cm by a vernier callipers with least count 0.01 cm, then

the radius r is recorded as

$$r = 2.25 \pm 0.01 \text{ cm}$$

Absolute uncertainty = Least count = ± 0.01 cm

$$\text{\%age uncertainty in } r = \frac{0.01\text{cm}}{2.25\text{ cm}} \times 100 = 0.4\%$$

Total percentage uncertainty in V = 3 x 0.4 = 1.2%

Thus volume $$V = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times 3.14 \times (2.25\text{ cm})^3$$

$$= 47.689 \text{ cm}^3 \text{ with } 1.2\% \text{ uncertainty}$$

Thus the result should be recorded as

$$V = 47.7 \pm 0.6 \text{ cm}^3$$

4. For uncertainty in the average value of many measurements.

(i) Find the average value of measured values.

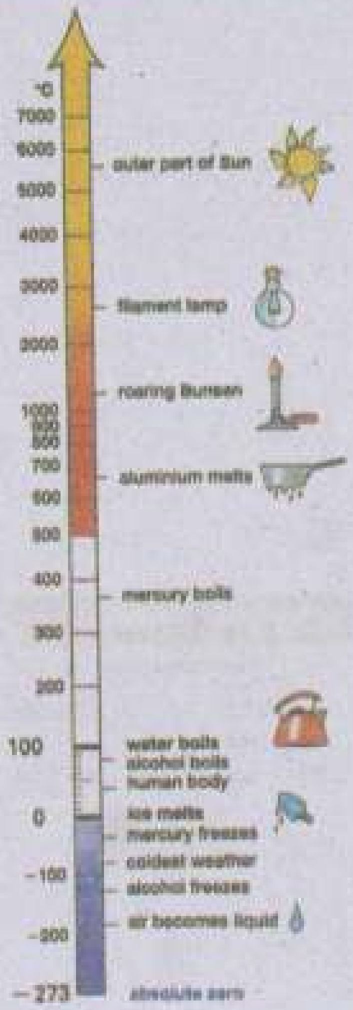
(ii) Find deviation of each measured value from the average value.

(iii) The mean deviation is the uncertainty in the average value.

For example, the six readings of the micrometer screw gauge to measure the diameter of a wire in mm are

1.20, 1.22, 1.23, 1.19, 1.22, 1.21.

Interesting Information



Some Specific Temperatures

Then $\text{Average} = \frac{1.20 + 1.22 + 1.23 + 1.19 + 1.22 + 1.21}{6}$

$= 1.21$ mm

The deviation of the readings, which are the difference without regards to the sign, between each reading and average value are 0.01, 0.01, 0.02, 0.02, 0.01, 0.

$$\text{Mean of deviations} = \frac{0.01 + 0.01 + 0.02 + 0.02 + 0.01 + 0}{6}$$

$= 0.01$ mm

Thus, likely uncertainty in the mean diametre 1.21 mm is 0.01 mm recorded as 1.21 ± 0.01 mm.

5. For the uncertainty in a timing experiment

The uncertainty in the time period of a vibrating body is found by dividing the least count of timing device by the number of vibrations. For example, the time of 30 vibrations of a simple pendulum recorded by a stopwatch accurate upto one tenth of a second is 54.6 s, the period

$$T = \frac{54.6\text{ s}}{30} = 1.82\text{ s with uncertainty } \frac{0.1\text{s}}{30} = 0.003\text{ s}$$

Thus, period T is quoted as $T = 1.82 \pm 0.003$ s

Hence, it is advisable to count large number of swings to reduce timing uncertainty.

For your information

Travel time of light

Moon to Earth	1 min 20s
Sun to Earth	8 min 20s
Pluto to Earth	5 h 20s

Example 1.1: The length, breadth and thickness of a sheet are 3.233m, 2.105 m and 1.05 cm respectively. Calculate the volume of the sheet correct upto the appropriate significant digits.

Solution: Given length $l = 3.233$ m

Breadth $b = 2.105$m

Thickness $h = 1.05\text{cm} = 1.05 \times 10^{-2}$ m

Volume $V = l \times b \times h$

$= 3.233\text{m} \times 2.105\text{m} \times 1.05 \times 10^{-2}\text{m}$

$= 7.14573825 \times 10^{-2}\ m^3$

As the factor 1.05 cm has minimum number of significant figures equal to three, therefore, volume is recorded upto 3 significant figures, hence, $V = 7.15 \times 10^{-2} m^3$

Example 1.2: The mass of a metal box measured by a lever balance is 2.2 kg. Two silver coins of masses 10.01 g and 10.02 g measured by a beam balance are added to it. What is now the total mass of the box correct upto the appropriate precision.

Solution: Total mass when silver coins are added to box

$= 2.2\ kg + 0.01001\ kg + 0.01002\ kg$

$= 2.22003\ kg$

Since least precise is 2.2 kg, having one decimal place, hence total mass should be to one decimal place which is the appropriate precision. Thus the total mass = 2.2 kg.

Example 1.3: The diameter and length of a metal cylinder measured with the help of vernier callipers of least count 0.01 cm are 1.22 cm and 5.35 cm. Calculate the volume *V* of the cylinder and uncertainty in it.

Solution: Given data is

Diameter $d = 1.22$ cm with least count 0.01 cm

Length $l = 5.35$ cm with least count 0.01 cm

Absolute uncertainty in length = 0.01 cm

$$\text{\%age uncertainty in length} = \frac{0.01\ cm}{5.35\ cm} \times 100 = 0.2\%$$

Absolute uncertainty in diameter = 0.01 cm

$$\text{\%age uncertainty in diameter} = \frac{0.01\ cm}{1.22\ cm} \times 100 = 0.8\%$$

As volume is $$V = \frac{\pi d^2 l}{4}$$

For Your Information

Atomic Clock

The cesium atomic frequency standard at the National Institute of Standards and Technology in Colorado (USA). It is the primary standard for the unit of time.

$\therefore$ total uncertainty in $V = 2$ (%age uncertainty in diameter)

+ (%age uncertainty in length)

$= 2 \times 0.8 + 0.2 = 1.8\%$

Then $V = \dfrac{3.14 \times (1.22\text{cm})^2 \times 5.35\text{ cm}}{4} = 6.2509079\text{ cm}^3$ with 1.8% uncertainty

Thus $V = (6.2 \pm 0.1)\text{ cm}^3$

Where 6.2 cm^3 is calculated volume and 0.1cm^3 is the uncertainty in it.

1.8 DIMENSIONS OF PHYSICAL QUANTITIES

Each base quantity is considered a dimension denoted by a specific symbol written within square brackets. It stands for the qualitative nature of the physical quantity. For example, different quantities such as length, breadth, diameter, light year which are measured in metre denote the same dimension and has the dimension of length [L]. Similarly the mass and time dimensions are denoted by [M] and [T], respectively. Other quantities that we measure have dimension which are combinations of these dimensions. For example, speed is measured in metres per second. This will obviously have the dimensions of length divided by time. Hence we can write.

$$\text{Dimensions of speed} = \frac{\text{Dimension of length}}{\text{Dimension of time}}$$

$$[v] = \frac{[L]}{[T]} = [L][T^{-1}] = [LT^{-1}]$$

Similarly the dimensions of acceleration are

$$[a] = [L][T^{-2}] = [LT^{-2}]$$

and that of force are

$$[F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$$

Using the method of dimensions called the dimensional analysis, we can check the correctness of a given formula or an equation and can also derive it. Dimensional analysis makes use of the fact that expression of the dimensions can be manipulated as algebraic quantities.

(i) Checking the homogeneity of physical equation

In order to check the correctness of an equation, we are to show that the dimensions of the quantities on both sides of the equation are the same, irrespective of the form of the formula. This is called the principle of homogeneity of dimensions.

Example 1.4: Check the correctness of the relation $v = \sqrt{\frac{F \times l}{m}}$ where v is the speed of transverse wave on a stretched string of tension F, length l and mass m.

Solution:

Dimensions of L.H.S. of the equation = $[v] = [LT^{-1}]$

Dimensions of R.H.S. of the equation = $([F] \times [l] \times [m^{-1}])^{1/2}$

$$= ([MLT^{-2}] \times [L] \times [M^{-1}])^{1/2} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$$

Since the dimensions of both sides of the equation are the same, equation is dimensionally correct.

(ii) Deriving a possible formula

The success of this method for deriving a relation for a physical quantity depends on the correct guessing of various factors on which the physical quantity depends.

Example 1.5: Derive a relation for the time period of a simple pendulum (Fig. 1.2) using dimensional analysis. The various possible factors on which the time period T may depend are :

i) Length of the pendulum (l)

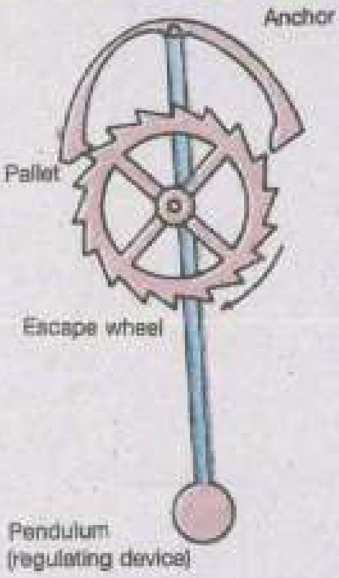
ii) Mass of the bob (m)

iii) Angle θ which the thread makes with the vertical

iv) Acceleration due to gravity (g)

Fig. 1.2

Do You Know?



The device which made the pendulum clock practical.

Solution:

The relation for the time period T will be of the form

$$T \propto m^a \times l^b \times \theta^c \times g^d$$

or $$T = \text{constant } m^a\, l^b\, \theta^c\, g^d \qquad \ldots\ldots\ldots \quad (1.1)$$

where we have to find the values of powers a, b, c and d.

Writing the dimensions of both sides we get

$$[T] = \text{constant} \times [M]^a [L]^b [LL^{-1}]^c [LT^{-2}]^d$$

Comparing the dimensions on both sides we have

$$[T] = [T]^{-2d}$$
$$[M]^0 = [M]^a$$
$$[L]^0 = [L]^{b+d+c-c}$$

Equating powers on both the sides we get

$$-2d = 1 \qquad \text{or} \qquad d = -\frac{1}{2}$$

$$a = 0 \qquad \text{and} \qquad b + d = 0$$

or $$b = -d = \frac{1}{2} \qquad \text{and} \qquad \theta = [L\,L^{-1}]^c = [L^0]^c = 1$$

Substituting the values of a, b, θ and d in Eq. 1.1

$$T = \text{constant} \times m^0 \times l^{½} \times 1 \times g^{-½}$$

Or $$T = \text{constant} \sqrt{\frac{l}{g}}$$

The numerical value of the constant cannot be determined by dimensional analysis, however, it can be found by experiments.

Example 1.6: Find the dimensions and hence, the SI units of coefficient of viscosity η in the relation of Stokes' law for the drag force F for a spherical object of radius r moving with velocity v given as $F = 6\ \pi\eta r v$

Solution: 6π is a number having no dimensions. It is not accounted in dimensional analysis. Then

$$[F] = [\eta r v]$$

or $$[\eta] = \frac{[F]}{[r][v]}$$

Substituting the dimensions of *F*, *r*, and *v* in R.H.S.

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

or $$[\eta] = [ML^{-1}T^{-1}]$$

Thus, the SI unit of coefficient of viscosity is kg m^{-1} s^{-1}

SUMMARY

- Physics is the study of entire Physical World.
- The most basic quantities that can be used to describe the Physical World are mass, length and time. All other quantities, called derived quantities, can be described in terms of some combinations of the base quantities.
- The internationally adopted system of units used by all the scientists and almost all the countries of the World is International System (SI) of Units. It consists of seven base units, two supplementary units and a number of derived units.
- Errors due to incorrect design or calibrations of the measuring device are called systematic errors. Random errors are due to unknown causes and fluctuations in the quantity being measured.
- The accuracy of a measurement is the extent to which systematic error make a measured value differ from its true value.
- The accuracy of a measurement can be indicated by the number of significant figures, or by a stated uncertainty.
- The significant figures or digits in a measured or calculated quantity are those digits that are known to be reasonably reliable.
- The result of multiplication or division has no more significant figures than any factor in the input data. Round off your calculator result to correct number of digits.
- In case of addition or subtraction the precision of the result can be only as great as the least precise term added or subtracted.
- Each basic measurable physical property represented by a specific symbol written within square brackets is called a dimension. All other physical quantities can be derived as combinations of the basic dimensions.
- Equations must be dimensionally consistent. Two terms can be added only when they have the same dimensions.

QUESTIONS

1.1 Name several repetitive phenomenon occurring in nature which could serve as reasonable time standards.

1.2 Give the drawbacks to use the period of a pendulum as a time standard.

1.3 Why do we find it useful to have two units for the amount of substance, the kilogram and the mole?

1.4 Three students measured the length of a needle with a scale on which minimum division is 1mm and recorded as (i) 0.2145 m (ii) 0.21 m (iii) 0.214m. Which record is correct and why?

1.5 An old saying is that "A chain is only as strong as its weakest link". What analogous statement can you make regarding experimental data used in a computation?

1.6 The period of simple pendulum is measured by a stop watch. What type of errors are possible in the time period?

1.7 Does a dimensional analysis give any information on constant of proportionality that may appear in an algebraic expression? Explain.

1.8 Write the dimensions of (i) Pressure (ii) Density

1.9 The wavelength λ of a wave depends on the speed v of the wave and its frequency f. Knowing that

$$[\lambda]=[L], \qquad [v]=[LT^{-1}] \qquad \text{and} \qquad [f]=[T^{-1}]$$

Decide which of the following is correct, $f = v\lambda$ or $f = \dfrac{v}{\lambda}$.

NUMERICAL PROBLEMS

1.1 A light year is the distance light travels in one year. How many metres are there in one light year: (speed of light = $3.0 \times 10^8\ ms^{-1}$).

(Ans: 9.5×10^{15}m)

1.2 a) How many seconds are there in 1 year?

b) How many nanoseconds in 1 year?

c) How many years in 1 second?

[Ans. (a) 3.1536×10^7s, (b) 3.1536×10^{16}ns (c) 3.1×10^{-8} yr]

1.3 The length and width of a rectangular plate are measured to be 15.3 cm and 12.80 cm, respectively. Find the area of the plate.

(Ans: 196 cm^2)

1.4 Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

(Ans: 19.4 kg)

1.5 Find the value of 'g' and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements made during an experiment

Length of simple pendulum l = 100 cm.

Time for 20 vibrations = 40.2 s

Length was measured by a metre scale of accuracy upto 1 mm and time by stop watch of accuracy upto 0.1 s.

(Ans: 9.76 ± 0.06 ms^{-2})

1.6 What are the dimensions and units of gravitational constant G in the formula $F = G\frac{m_1 m_2}{r^2}$

(Ans: $[M^{-1}L^3T^{-2}]$, Nm^{2}kg^{-2})

1.7 Show that the expression $v_f = v_i + at$ is dimensionally correct, where v_i is the velocity at $t = 0$, a is acceleration and v_f is the velocity at time t.

1.8 The speed v of sound waves through a medium may be assumed to depend on (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by the method of dimensions, the formula for the speed of sound.

(Ans: v = Constant $\sqrt{\frac{E}{\rho}}$)

1.9 Show that the famous "Einstein equation" $E = mc^2$ is dimensionally consistent.

1.10 Suppose, we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v?

(Ans: n = -1, m = 2)

Chapter 2

VECTORS AND EQUILIBRIUM

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand and use rectangular coordinate system.
2. Understand the idea of unit vector, null vector and position vector.
3. Represent a vector as two perpendicular components (rectangular components).
4. Understand the rule of vector addition and extend it to add vectors using rectangular components.
5. Understand multiplication of vectors and solve problems.
6. Define the moment of force or torque.
7. Appreciate the use of the torque due to a force.
8. Show an understanding that when there is no resultant force and no resultant torque, a system is in equilibrium.
9. Appreciate the applications of the principle of moments.
10. Apply the knowledge gained to solve problems on statics.

Physical quantities that have both numerical and directional properties are called vectors. This chapter is concerned with the vector algebra and its applications in problems of equilibrium of forces and equilibrium of torques.

2.1 BASIC CONCEPTS OF VECTORS

(i) Vectors

As we have studied in school physics, there are some physical quantities which require both magnitude and direction for their complete description, such as velocity, acceleration and force. They are called vectors. In books, vectors are usually denoted by bold face characters such as **A**, **d**, **r** and **v** while in handwriting, we put an arrowhead over the letter e.g. $\vec{d}$. If we wish to refer only to the magnitude of a vector **d** we use light face type such as *d*.

A vector is represented graphically by a directed line segment with an arrowhead. The length of the line segment, according to a chosen scale, corresponds to the magnitude of the vector.

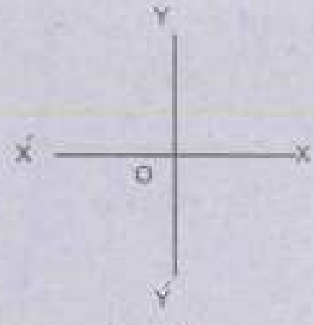


Fig. 2.1(a)

(ii) Rectangular coordinate system

Two reference lines drawn at right angles to each other as shown in Fig. 2.1 (a) are known as coordinate axes and their point of intersection is known as origin. This system of coordinate axes is called Cartesian or rectangular coordinate system.

One of the lines is named as x-axis, and the other the y-axis. Usually the x-axis is taken as the horizontal axis, with the positive direction to the right, and the y-axis as the vertical axis with the positive direction upward.

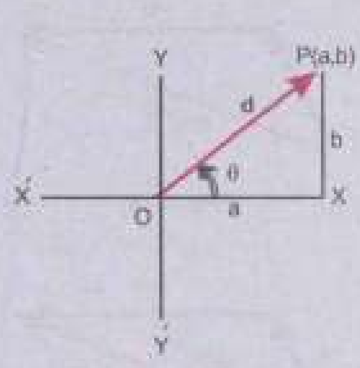


Fig. 2.1(b)

The direction of a vector in a plane is denoted by the angle which the representative line of the vector makes with positive x-axis in the anti-clock wise direction, as shown in Fig 2.1 (b). The point P shown in Fig 2.1 (b) has coordinates (a,b). This notation means that if we start at the origin, we can reach P by moving 'a' units along the positive x-axis and then 'b' units along the positive y-axis.

The direction of a vector in space requires another axis which is at right angle to both x and y axes, as shown in Fig 2.2 (a). The third axis is called z-axis.

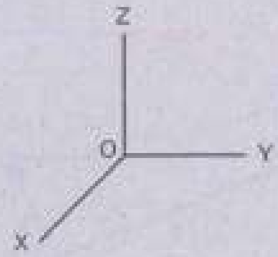


Fig. 2.2(a)

The direction of a vector in space is specified by the three angles which the representative line of the vector makes with x, y and z axes respectively as shown in Fig 2.2 (b). The point P of a vector **A** is thus denoted by three coordinates (a, b, c).

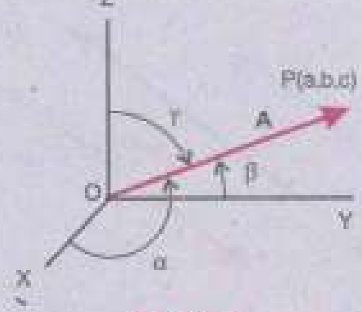


Fig. 2.2(b)

(iii) Addition of Vectors

Given two vectors **A** and **B** as shown in Fig 2.3 (a), their sum is obtained by drawing their representative lines in such a way that tail of vector **B** coincides with the head of the vector **A**. Now if we join the tail of **A** to the head of **B**, as shown in

Fig. 2.3(a)

the Fig. 2.3 (b), the line joining the tail of **A** to the head of **B** will represent the vector sum (**A**+**B**) in magnitude and direction. The vector sum is also called resultant and is indicated by **R**. Thus **R** = **A**+**B**. This is known as head to tail rule of vector addition. This rule can be extended to find the sum of any number of vectors. Similarly the sum **B** + **A** is illustrated by black lines in Fig 2.3 (c). The answer is same resultant **R** as indicated by the red line. Therefore, we can say that

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad \text{..........} \qquad (2.1)$$

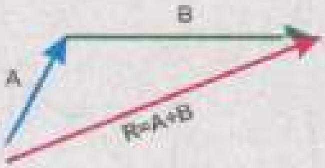


Fig. 2.3(b)

So the vector addition is said to be commutative. It means that when vectors are added, the result is the same for any order of addition.

(iv) Resultant Vector

The resultant of a number of vectors of the same kind–force vectors for example, is that single vector which would have the same effect as all the original vectors taken together.

Fig. 2.3(c)

(v) Vector Subtraction

The subtraction of a vector is equivalent to the addition of the same vector with its direction reversed. Thus, to subtract vector **B** from vector **A**, reverse the direction of **B** and add it to **A**, as shown in Fig. 2.3 (d).

A - **B** = **A** + (-**B**) where (-**B**) is negative vector of **B**

(vi) Multiplication of a Vector by a Scalar

Fig. 2.3(d)

The product of a vector **A** and a number n > 0 is defined to be a new vector n**A** having the same direction as **A** but a magnitude n times the magnitude of **A** as illustrated in Fig. 2.4. If the vector is multiplied by a negative number, then its direction is reversed.

In the event that n represents a scalar quantity, the product n**A** will correspond to a new physical quantity and the dimensions of the resulting vector will be the product of the dimensions of the two quantities which were multiplied together. For example, when velocity is multiplied by scalar mass *m*, the product is a new vector quantity called momentum having the dimensions as those of mass and velocity.

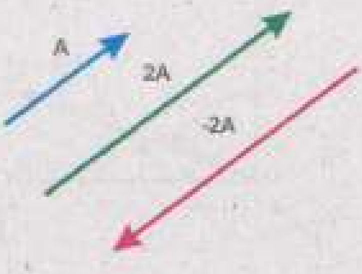


Fig. 2.4

(vii) Unit Vector

A unit vector in a given direction is a vector with magnitude one in that direction. It is used to represent the direction of a vector.

A unit vector in the direction of **A** is written as $\hat{\mathbf{A}}$, which we read as 'A hat', thus

$$\mathbf{A} = A\hat{\mathbf{A}}$$

$$\hat{\mathbf{A}} = \frac{\mathbf{A}}{A} \qquad \text{..........} \qquad (2.2)$$

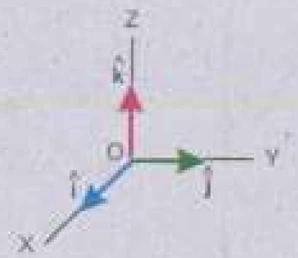


Fig. 2.5(a)

The direction along x, y and z axes are generally represented by unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ respectively (Fig. 2.5 a). The use of unit vectors is not restricted to Cartesian coordinate system only. Unit vectors may be defined for any direction. Two of the more frequently used unit vectors are the vector $\hat{\mathbf{r}}$ which represents the direction of the vector **r** (Fig. 2.5 b) and the vector $\hat{\mathbf{n}}$ which represents the direction of a normal drawn on a specified surface as shown in Fig 2.5 (c).

Fig. 2.5(b)

(viii) Null Vector

Null vector is a vector of zero magnitude and arbitrary direction. Forexample, the sum of a vector and its negative vector is a null vector.

$$\mathbf{A} + (-\mathbf{A}) = \mathbf{0} \qquad \text{..........} \qquad (2.3)$$

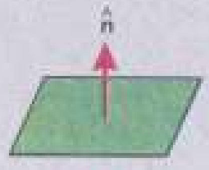


Fig. 2.5(c)

(ix) Equal Vectors

Two vectors **A** and **B** are said to be equal if they have the same magnitude and direction, regardless of the position of their initial points.

This means that parallel vectors of the same magnitude are equal to each other.

(x) Rectangular Components of a Vector

A component of a vector is its effective value in a given direction. A vector may be considered as the resultant of its component vectors along the specified directions. It is usually convenient to resolve a vector into components along mutually perpendicular directions. Such components are called rectangular components.

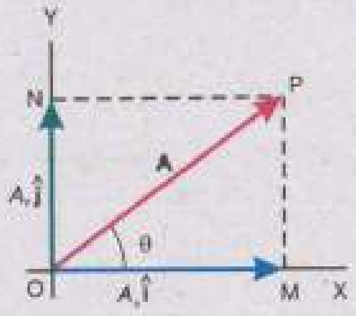


Fig. 2.6

Let there be a vector **A** represented by OP making angle θ with the x-axis. Draw projection OM of vector OP on x-axis and projection ON of vector OP on y-axis as shown in Fig.2.6. Projection OM being along x-direction is represented by $A_x\hat{i}$ and projection ON = MP along y-direction is represented by $A_y\hat{j}$. By head and tail rule

$$\mathbf{A} = A_x\hat{i} + A_y\hat{j} \quad \text{..........} \quad (2.4)$$

Thus $A_x\hat{i}$ and $A_y\hat{j}$ are the components of vector **A**. Since these are at right angle to each other, hence, they are called rectangular components of **A**. Considering the right angled triangle OMP, the magnitude of $A_x\hat{i}$ or x-component of **A** is

$$A_x = A\cos\theta \quad \text{..........} \quad (2.5)$$

And that of $A_y\hat{j}$ or y-component of **A** is

$$A_y = A\sin\theta \quad \text{..........} \quad (2.6)$$

(xi) Determination of a Vector from its Rectangular Components

If the rectangular components of a vector, as shown in Fig. 2.6, are given, we can find out the magnitude of the vector by using Pythagorean theorem.

In the right angled Δ OMP,

$$OP^2 = OM^2 + MP^2$$

or
$$A^2 = A_x^2 + A_y^2 \quad \text{..........} \quad (2.7)$$

or
$$A = \sqrt{A_x^2 + A_y^2}$$

and direction θ is given by $\tan\theta = \frac{MP}{OM} = \frac{A_y}{A_x}$

or
$$\theta = \tan^{-1}\frac{A_y}{A_x} \quad \text{..........} \quad (2.8)$$

(xii) Position Vector

The position vector **r** is a vector that describes the location of a point with respect to the origin. It is represented by a straight line drawn in such a way that its tail coincides with the origin and the head with point P (a,b) as shown in Fig. 2.7(a). The projections of position vector **r** on the x and y axes are the coordinates a and b and they are the rectangular components of the vector **r**. Hence

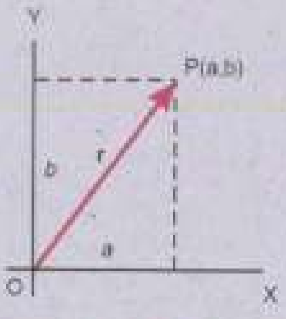


Fig. 2.7(a)

$$\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} \quad \text{and} \quad r = \sqrt{a^2 + b^2} \quad \text{.........} \quad (2.9)$$

In three dimensional space, the position vector of a point P (a,b,c) is shown in Fig. 2.7 (b) and is represented by

$$\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}} \text{ and } r = \sqrt{a^2 + b^2 + c^2} \quad \text{.........} \quad (2.10)$$

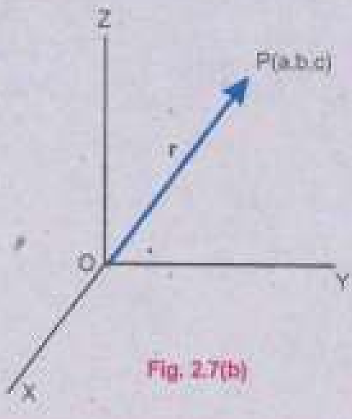


Fig. 2.7(b)

Example 2.1: The positions of two aeroplanes at any instant are represented by two points A (2, 3, 4) and B(5, 6, 7) from an origin O in km as shown in Fig. 2.8.
(i) What are their position vectors?
(ii) Calculate the distance between the two aeroplanes.

Solution: (i) A position vector **r** is given by

$$\mathbf{r} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$$

Thus position vector of first aeroplane A is

$$\mathbf{OA} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

And position vector of the second aeroplane B is

$$\mathbf{OB} = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$$

By head and tail rule

$$\mathbf{OA} + \mathbf{AB} = \mathbf{OB}$$

Therefore, the distance between two aeroplanes is given by

$$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 7\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$= (3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

Magnitude of vector **AB** is the distance between the position of two aeroplanes which is then:

$$AB = \sqrt{(3\text{km})^2 + (3\text{km})^2 + (3\text{km})^2} = 5.2\text{ km}$$

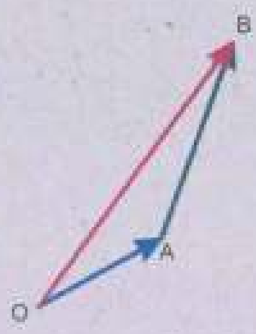


Fig. 2.8

2.2 VECTOR ADDITION BY RECTANGULAR COMPONENTS

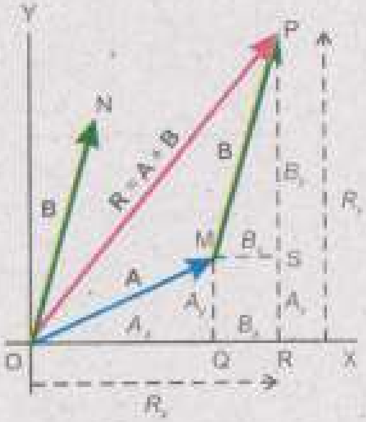


Fig. 2.9

Let **A** and **B** be two vectors which are represented by two directed lines OM and ON respectively. The vector **B** is added to **A** by the head to tail rule of vector addition (Fig 2.9). Thus the resultant vector **R** = **A** + **B** is given, in direction and magnitude by the vector **OP**.

In the Fig 2.9 $\mathbf{A}_x$, $\mathbf{B}_x$ and $\mathbf{R}_x$ are the x components of the vectors **A**, **B** and **R** and their magnitudes are given by the lines OQ, MS, and OR respectively. But

$$OR = OQ + QR$$

or $$OR = OQ + MS$$

or $$R_x = A_x + B_x \quad \text{..........} \quad (2.11)$$

which means that the sum of the magnitudes of x-components of two vectors which are to be added, is equal to the x-component of the resultant. Similarly the sum of the magnitudes of y-components of two vectors is equal to the magnitude of y-component of the resultant, that is

$$R_y = A_y + B_y \quad \text{..........} \quad (2.12)$$

Since R_x and R_y are the rectangular components of the resultant vector **R**, hence

$$\mathbf{R} = R_x\,\hat{i} + R_y\,\hat{j}$$

or $$\mathbf{R} = (A_x + B_x)\,\hat{i} + (A_y + B_y)\,\hat{j}$$

The magnitude of the resultant vector **R** is thus given as

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \quad \text{..........} \quad (2.13)$$

and the direction of the resultant vector is determined from

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{(A_y + B_y)}{(A_x + B_x)}$$

and $$\theta = \tan^{-1}\frac{(A_y + B_y)}{(A_x + B_x)} \quad \ldots\ldots\ldots \quad (2.14)$$

Similarly for any number of coplanar vectors **A**, **B**, **C**...., we can write

$$R = \sqrt{(A_x + B_x + C_x + \ldots)^2 + (A_y + B_y + C_y + \ldots)^2} \quad \ldots\ldots \quad (2.15)$$

and $$\theta = \tan^{-1}\frac{(A_y + B_y + C_y + \ldots.)}{(A_x + B_x + C_x + \ldots.)} \quad \ldots\ldots\ldots \quad (2.16)$$

The vector addition by rectangular components consists of the following steps.

i) Find x and y components of all given vectors.

ii) Find x-component R_x of the resultant vector by adding the x-components of all the vectors.

iii) Find y-component R_y of the resultant vector by adding the y-components of all the vectors.

iv) Find the magnitude of resultant vector **R** using

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$

v) Find the direction of resultant vector **R** by using

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

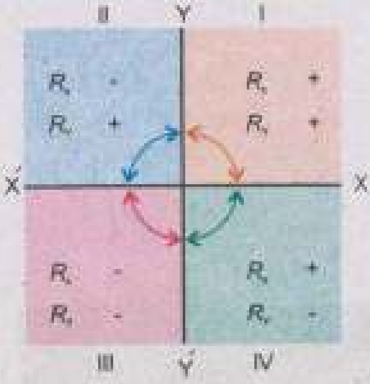
where θ is the angle, which the resultant vector makes with positive x-axis. The signs of R_x and R_y determine the quadrant in which resultant vector lies. For that purpose proceed as given below.

Irrespective of the sign of R_x and R_y, determine the value of $\tan^{-1}\frac{R_y}{R_x} = \phi$ from the calculator or by consulting trigonometric tables. Knowing the value of ϕ, angle θ is determined as follows.

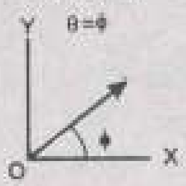
Do You Know?

The Chinese acrobats in a incredible balancing act are equilibrium.

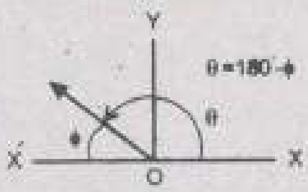
Table 2.1



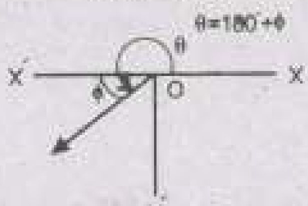
1st quadrant



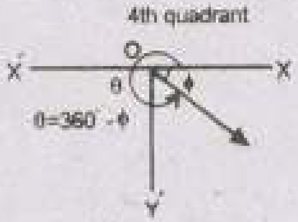
2nd quadrant



3rd quadrant



4th quadrant



a) If both R_x and R_y are positive, then the resultant lies in the first quadrant and its direction is $\theta = \phi$.

b) If R_x is –ive and R_y is +ive, the resultant lies in the second quadrant and its direction is $\theta = 180^\circ - \phi$.

c) If both R_x and R_y are –ive, the resultant lies in the third quadrant and its direction is $\theta = 180^\circ + \phi$.

d) If R_x is positive and R_y is negative, the resultant lies in the fourth quadrant and its direction is $\theta = 360^\circ - \phi$.

Example 2.2: Two forces of magnitude 10 N and 20 N act on a body in directions making angles 30° and 60° respectively with x-axis. Find the resultant force.

Solution:

Step (i) **x-components**

The x-component of the first force $= F_{1x} = F_1 \cos 30^\circ$

$= 10\ \text{N} \times 0.866 = 8.66\ \text{N}$

The x-component of second force $= F_{2x} = F_2 \cos 60^\circ$

$= 20\ \text{N} \times 0.5 = 10\ \text{N}$

y-components

The y-component of the first force $= F_{1y} = F_1 \sin 30^\circ$

$= 10\ \text{N} \times 0.5 = 5\ \text{N}$

The y-component of second force $= F_{2y} = F_2 \sin 60^\circ$

$= 20\ \text{N} \times 0.866 = 17.32\ \text{N}$

Step (ii)

The magnitude of x component F_x of the resultant force **F**

$$F_x = F_{1x} + F_{2x}$$

$$F_x = 8.66\ \text{N} + 10\ \text{N} = 18.66\ \text{N}$$

Step (iii)

The magnitude of y component F_y of the resultant force **F**

$$F_y = F_{1y} + F_{2y}$$

$$F_y = 5\ \text{N} + 17.32\ \text{N} = 22.32\ \text{N}$$

Step (iv)

The magnitude F of the resultant force **F**

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(18.66\,N)^2 + (22.32\,N)^2} = 29\ N$$

Step (v)

If the resultant force **F** makes an angle θ with the x-axis then

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{22.32N}{18.66N} = \tan^{-1} 1.196 = 50°.$$

Example 2.3: Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Solution: Let θ be the angle between two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1$ is along x-axis. Then x-component of their resultant will be

$$R_x = F_1 \cos 0° + F_2 \cos\theta$$

$$R_x = F_1 + F_2 \cos\theta$$

And y-component of their resultant is

$$R_y = F_1 \sin 0° + F_2 \sin\theta$$

$$R_y = F_2 \sin\theta$$

The resultant R is given by $R^2 = R_x^2 + R_y^2$

As $R = F_1 = F_2 = F$

Hence $F^2 = (F + F\cos\theta)^2 + (F\sin\theta)^2$

Or $0 = 2F^2\cos\theta + F^2(\cos^2\theta + \sin^2\theta)$

Or $0 = 2F^2\cos\theta + F^2$

Or $\cos\theta = -0.5$

Or $\theta = \cos^{-1}(-0.5) = 120°$

Point to Ponder

Why do you keep your legs far apart when you have to stand in the aisle of a bumpy-riding bus?

2.3 PRODUCT OF TWO VECTORS

There are two types of vector multiplications. The product of these two types are known as scalar product and vector product. As the name implies, scalar product of two vector quantities is a scalar quantity, while vector product of two vector quantities is a vector quantity.

Scalar or Dot Product

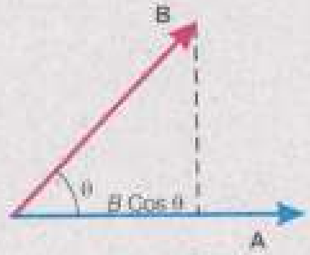


Fig. 2.10 (a)

The scalar product of two vectors **A** and **B** is written as **A** . **B** and is defined as

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta \qquad \text{..........} \qquad (2.17)$$

where A and B are the magnitudes of vectors **A** and **B** and θ is the angle between them.

For physical interpretation of dot product of two vectors **A** and **B**, these are first brought to a common origin (Fig. 2.10 a),

then, $\mathbf{A}.\mathbf{B} = (A)$ (projection of **B** on **A**)

or

$\mathbf{A}.\mathbf{B} = A$ (magnitude of component of **B** in the direction of **A**)

$= A\,(B \cos \theta) = AB \cos \theta$

Similarly $\quad \mathbf{B}.\mathbf{A} = B\,(A \cos \theta) = BA \cos \theta$

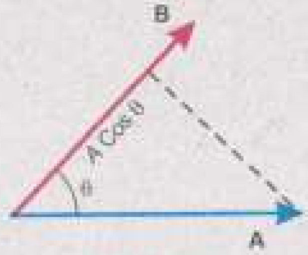


Fig. 2.10 (b)

We come across this type of product when we consider the work done by a force **F** whose point of application moves a distance d in a direction making an angle θ with the line of action of **F**, as shown in Fig. 2.11.

Work done = (effective component of force in the direction of motion) x distance moved

$= (F \cos \theta)\, d = Fd \cos \theta$

Fig. 2.11

Using vector notation

$$\mathbf{F}.\mathbf{d} = Fd \cos \theta = \text{work done}$$

Characteristics of Scalar Product

1. Since $\mathbf{A}.\mathbf{B} = AB \cos \theta$ and $\mathbf{B}.\mathbf{A} = BA \cos \theta = AB \cos \theta$, hence, $\mathbf{A}.\mathbf{B} = \mathbf{B}.\mathbf{A}$. The order of multiplication is irrelevant. In other words, scalar product is commutative.
2. The scalar product of two mutually perpendicular vectors is zero. $\mathbf{A}.\mathbf{B} = AB \cos 90^{\circ} = 0$

In case of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, since they are mutually perpendicular, therefore,

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0 \quad \text{..........} \quad (2.18)$$

3. The scalar product of two parallel vectors is equal to the product of their magnitudes. Thus for parallel vectors ($\theta = 0^\circ$)

$$\mathbf{A.B} = AB\cos 0^\circ = AB$$

In case of unit vectors

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \quad \text{..........} \quad (2.19)$$

and for antiparallel vectors ($\theta = 180^\circ$)

$$\mathbf{A.B} = AB\cos 180^\circ = -AB$$

4. The self product of a vector **A** is equal to square of its magnitude.

$$\mathbf{A.A} = AA\cos 0^\circ = A^2$$

5. Scalar product of two vectors **A** and **B** in terms of their rectangular components

$$\mathbf{A.B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

or $$\mathbf{A.B} = A_xB_x + A_yB_y + A_zB_z \quad \text{..........} \quad (2.20)$$

Equation 2.17 can be used to find the angle between two vectors: Since,

$$\mathbf{A.B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

Therefore, $$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} \quad \text{.....} \quad (2.21)$$

What should You do?

You are falling off the edge. What should you do to avoid falling?

Example 2.4: A force $\mathbf{F} = 2\hat{i} + 3\hat{j}$ units, has its point of application moved from point A(1,3) to the point B (5,7). Find the work done.

Solution: Position vector of point A is $\mathbf{r}_A = \hat{i} + 3\hat{j}$ and that of point B is $\mathbf{r}_B = 5\hat{i} + 7\hat{j}$

Displacement $\mathbf{d} = \mathbf{r}_B - \mathbf{r}_A = (5-1)\,\hat{i} + (7-3)\,\hat{j} = 4\hat{i} + 4\hat{j}$

$$\text{Work done} = \mathbf{F.d} = (2\hat{i} + 3\hat{j}).(4\hat{i} + 4\hat{j})$$

$$= 8 + 12 = 20 \text{ units}$$

Example 2.5: Find the projection of vector $\mathbf{A} = 2\hat{i} - 8\hat{j} + \hat{k}$ in the direction of the vector $\mathbf{B} = 3\hat{i} - 4\hat{j} - 12\hat{k}$.

Solution: If θ is the angle between **A** and **B**, then $A\cos\theta$ is the required projection.

By definition $\mathbf{A.B} = AB\cos\theta$

$$A\cos\theta = \frac{\mathbf{A.B}}{B} = \mathbf{A}.\hat{\mathbf{B}}$$

Where $\hat{\mathbf{B}}$ is the unit vector in the direction of **B**

Now $B = \sqrt{3^2 + (-4)^2 + (-12)^2} = 13$

Therefore, $\hat{\mathbf{B}} = \dfrac{(3\hat{i} - 4\hat{j} - 12\hat{k})}{13}$

The projection of **A** on $\mathbf{B} = (2\hat{i} - 8\hat{j} + \hat{k}).\dfrac{(3\hat{i} - 4\hat{j} - 12\hat{k})}{13}$

$$= \frac{(2)\,(3) + (-8)\,(-4) + 1\,(-12)}{13} = \frac{26}{13} = 2$$

Vector or Cross Product

The vector product of two vectors **A** and **B**, is a vector which is defined as

$$\mathbf{A} \times \mathbf{B} = AB\sin\theta\,\hat{\mathbf{n}} \quad \text{..........} \quad (2.22)$$

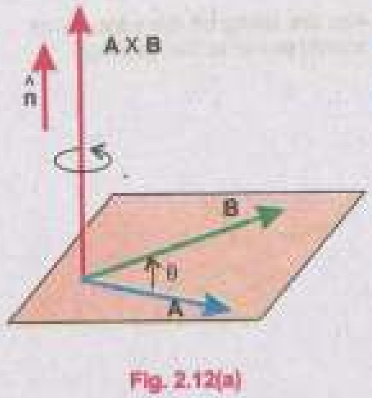


Fig. 2.12(a)

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane containing **A** and **B** as shown in Fig. 2.12 (a). Its direction can be determined by right hand rule. For that purpose, place together the tails of vectors **A** and **B** to define the plane of vectors **A** and **B.** The direction of the product vector is perpendicular to this plane. Rotate the first vector **A** into **B** through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector will be along the erect thumb, as shown in the Fig 2.12 (b). Because of this direction rule, **B x A** is a vector opposite in sign to **A x B**. Hence,

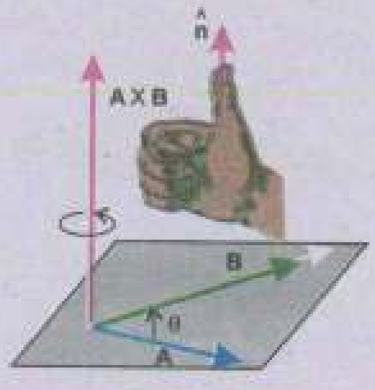


Fig. 2.12(b)

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A} \quad \text{..........} \quad (2.23)$$

Characteristics of Cross Product

1. Since **A x B** is not the same as **B x A**, the cross product is non commutative.

2. The cross product of two perpendicular vectors has maximum magnitude $\mathbf{A} \times \mathbf{B} = AB \sin 90° \hat{n} = AB\,\hat{n}$

   In case of unit vectors, since they form a right handed system and are mutually perpendicular Fig. 2.5 (a)

   $$\hat{i} \times \hat{j} = \hat{k}\ ,\ \hat{j} \times \hat{k} = \hat{i}\ ,\ \hat{k} \times \hat{i} = \hat{j}$$

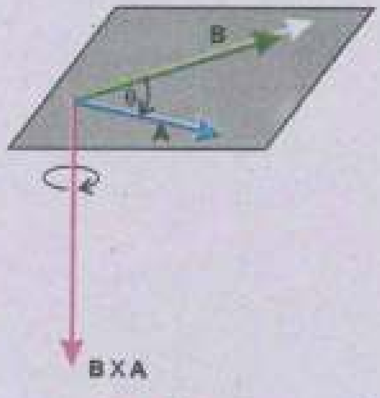


Fig. 2.12(c)

3. The cross product of two parallel vectors is null vector, because for such vectors θ = 0° or 180°. Hence

   $$\mathbf{A} \times \mathbf{B} = AB \sin 0° \hat{n} = AB \sin 180° \hat{n} = \mathbf{0}$$

   As a consequence $\mathbf{A} \times \mathbf{A} = \mathbf{0}$

$$\text{Also} \quad \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \mathbf{0} \quad \text{..........} \quad (2.24)$$

4. Cross product of two vectors **A** and **B** in terms of their rectangular components is :

   $$\mathbf{A} \times \mathbf{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\,\hat{i} + (A_zB_x - A_xB_z)\,\hat{j} + (A_xB_y - A_yB_x)\,\hat{k} \quad \text{..........} \quad (2.25)$$

Fig. 2.12(d)

The result obtained can be expressed for memory in determinant form as below:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

5. The magnitude of **A x B** is equal to the area of the parallelogram formed with **A** and **B** as two adjacent sides (Fig. 2.12 d).

Examples of Vector Product

i. When a force **F** is applied on a rigid body at a point whose position vector is **r** from any point of the axis about which the body rotates, then the turning effect of the force, called the torque **τ** is given by the vector product of **r** and **F**.

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

ii. The force on a particle of charge *q* and velocity **v** in a magnetic field of strength **B** is given by vector product.

$$\mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

2.4 TORQUE

The nut is easy to turn with a spanner.

It is easier still if the spanner has a long handle.

Fig. 2.13

We have already studied in school physics that a turning effect is produced when a nut is tightened with a spanner (Fig. 2.13). The turning effect increases when you push harder on the spanner. It also depends on the length of the spanner: the longer the handle of the spanner, the greater is the turning effect of an applied force. The turning effect of a force is called its moment or torque and its magnitude is defined as the product of force **F** and the perpendicular distance from its line of action to the pivot which is the point O around which the body (spanner) rotates. This distance OP is called moment arm l. Thus the magnitude of torque represented by τ is

$$\tau = l\,F \qquad \text{..........} \qquad (2.26)$$

When the line of action of the applied force passes through the pivot point, the value of moment arm l = 0, so in this case torque is zero.

We now consider the torque due to a force **F** acting on a rigid body. Let the force **F** acts on rigid body at point P whose position vector relative to pivot O is **r**. The force **F** can be resolved into two rectangular components, $F \sin\theta$ perpendicular to **r** and $F \cos\theta$ along the direction of **r** (Fig. 2.14 a). The torque due to $F \cos\theta$ about pivot O is zero as its line of action passes through point O. Therefore, the magnitude of torque due to **F** is equal to the torque due to $F \sin\theta$ only about O. It is given by

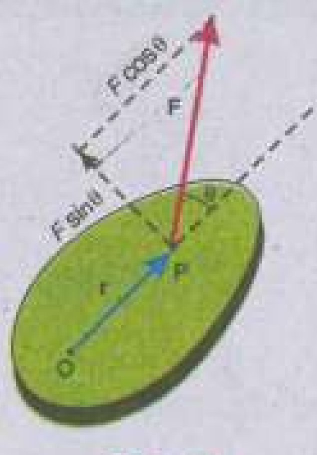


Fig. 2.14(a)

$$\tau = (F \sin\theta)\, r = r F \sin\theta \quad \text{.........} \quad (2.27)$$

Alternatively the moment arm l is equal to the magnitude of the component of **r** perpendicular to the line of action of **F** as illustrated in Fig. 2.14 (b). Thus

$$\tau = (r \sin\theta)\, F = rF \sin\theta \quad \text{..........} \quad (2.28)$$

where θ is the angle between **r** and **F**

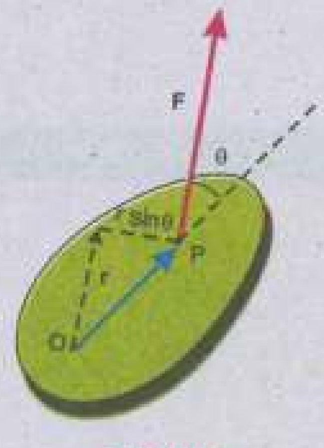


Fig. 2.14(b)

From Eq. 2.27 and Eq. 2.28 it can be seen that the torque can be defined by the vector product of position vector **r** and the force **F**, so

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

or $$\boldsymbol{\tau} = (r F \sin\theta)\, \hat{\mathbf{n}} \quad \text{..........} \quad (2.29)$$

Where $(rF \sin\theta)$ is the magnitude of the torque. The direction of the torque represented by $\hat{\mathbf{n}}$ is perpendicular to the plane containing **r** and **F** given by right hand rule for the vector product of two vectors.

The SI unit for torque is newton metre (N m).

Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration. Torque is the analogous of force for rotational motion. If the body is at rest or rotating with uniform angular velocity, the angular acceleration will be zero. In this case the torque acting on the body will be zero.

Point to Ponder

Do you think the rider in the above figure is really in danger? What if people below were removed?

Can You Do ?

Stand with one arm and the side of one foot pressed against a wall. Can you raise the other leg side ways? If not, then why not?

Example 2.6: The line of action of a force **F** passes through a point P of a body whose position vector in metre is $\hat{i}-2\hat{j}+\hat{k}$. If $\mathbf{F} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ (in newton), determine the torque about the point 'A' whose position vector (in metre) is $2\hat{i}+\hat{j}+\hat{k}$

Solution:

The position vector of point A = $\mathbf{r}_1 = 2\hat{i}+\hat{j}+\hat{k}$

The position vector of point P= $\mathbf{r}_2 = \hat{i}-2\hat{j}+\hat{k}$ relative to O,

The position vector of P relative to A is

$$\mathbf{AP} = \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

$$\mathbf{AP} = (\hat{i}-2\hat{j}+\hat{k}) - (2\hat{i}+\hat{j}+\hat{k}) = -\hat{i}-3\hat{j}$$

The torque about A = $\mathbf{r} \times \mathbf{F}$

$$= (-\hat{i}-3\hat{j}) \times (2\hat{i}-3\hat{j}+4\hat{k})$$

$$= -12\hat{i}+4\hat{j}+9\hat{k} \text{ N m}$$

2.5 EQUILIBRIUM OF FORCES

We have studied in school physics that if a body, under the action of a number of forces, is at rest or moving with uniform velocity, it is said to be in equilibrium.

First Condition of Equilibrium

A body at rest or moving with uniform velocity has zero acceleration. From Newton's Law of motion the vector sum of all forces acting on it must be zero.

This is known as the first condition of equilibrium. Using the mathematical symbol $\Sigma \mathbf{F}$ for the sum of all forces we can write

$$\Sigma \mathbf{F} = 0 \quad \ldots\ldots\ldots \quad (2.30)$$

In case of coplanar forces, this condition is expressed usually in terms of x and y components of the forces. We have studied that x-component of the resultant force **F** equals the sum of x-directed or x-components of all the forces acting on the body. Hence

$$\Sigma \mathbf{F}_x = 0 \qquad \text{..........} \qquad (2.31)$$

Similarly for the y-directed forces, the resultant of y-directed forces should be zero. Hence

$$\Sigma \mathbf{F}_y = 0 \qquad \text{..........} \qquad (2.32)$$

It may be noted that if the rightward forces are taken as positive then leftward forces are taken as negative. Similarly if upward forces are taken as positive then downward forces are taken as negative.

Interesting Application

A concurrent force system in equilibrium. The tension applied can be adjusted as desired.

Example 2.7: A load is suspended by two cords as shown in Fig. 2.15. Determine the maximum load that can be suspended at P, if maximum breaking tension of the cord used is 50 N.

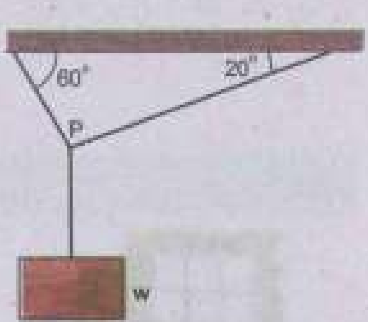


Fig. 2.15

Solution: For using conditions of equilibrium, all the forces acting at point P are shown by a force diagram as illustrated in Fig. 2.16 where **w** is assumed to be the maximum weight which can be suspended. The inclined forces can now be easily resolved along x and y directions.

Applying $\Sigma F_x = 0$

$T_2 \cos 20^\circ - T_1 \cos 60^\circ = 0$

Or $T_1 = 1.88\, T_2$

As $T_1 > T_2$ $\therefore$ T_1 has the maximum tension

If $T_1 = 50$ N, then $T_2 = 26.6$ N

Now applying $\Sigma F_y = 0$

$T_1 \sin 60^\circ + T_2 \sin 20^\circ - w = 0$

Putting the values

$50\text{ N} \times 0.866 + 26.6\text{ N} \times 0.34 = w$

or $w = 52$ N

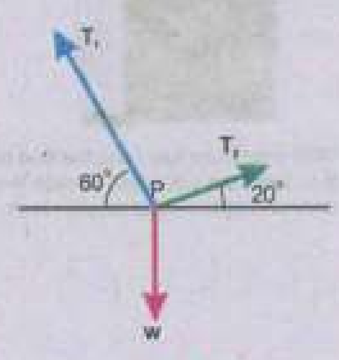


Fig. 2.16

2.6 EQUILIBRIUM OF TORQUES

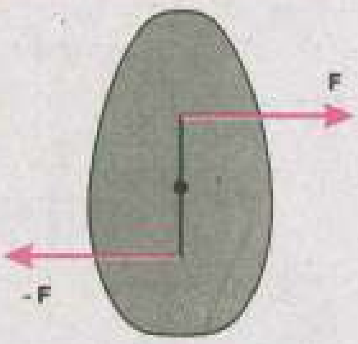


Fig. 2.17

Second Condition of Equilibrium

Let two equal and opposite forces act on a rigid body as shown in Fig. 2.17. Although the first condition of equilibrium is satisfied, yet it may rotate having clockwise turning effect. As discussed earlier, for angular acceleration to be zero, the net torque acting on the body should be zero. Thus for a body in equilibrium, the vector sum of all the torques acting on it about any arbitrary axis should be zero. This is known as second condition of equilibrium. Mathematically it is written as

$$\Sigma\tau = 0 \qquad \text{..........} \qquad (2.33)$$

By convention, the counter clockwise torques are taken as positive and clockwise torques as negative. An axis is chosen for calculating the torques. The position of the axis is quite arbitrary. Axis can be chosen anywhere which is convenient in applying the torque equation. A most helpful point of rotation is the one through which lines of action of several forces pass.

We are now in a position to state the complete requirements for a body to be in equilibrium, which are

(1) $\Sigma \mathbf{F} = 0$ i.e $\Sigma \mathbf{F}_x = 0$ and $\Sigma \mathbf{F}_y = 0$

(2) $\Sigma\tau = 0$

When 1st condition is satisfied, there is no linear acceleration and body will be in translational equilibrium. When 2nd condition is satisfied, there is no angular acceleration and body will be in rotational equilibrium.

For a body to be in complete equilibrium, both conditions should be satisfied, i.e., both linear acceleration and angular acceleration should be zero.

If a body is at rest, it is said to be in static equilibrium but if the body is moving with uniform velocity, it is said to be in dynamic equilibrium.

Can You Do?

With your nose touching the end of the door, put your feet astride the door and try to rise up on your toes.

We will restrict the applications of above mentioned conditions of equilibrium to situations in which all the forces lie in a common plane. Such forces are said to be coplanar. We will also assume that these forces lie in the xy-plane.

If there are more than one object in equilibrium in a given problem, one object is selected at a time to apply the conditions of equilibrium.

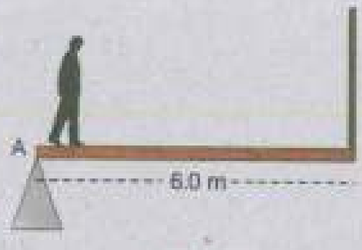


Fig. 2.18

Example 2.8: A uniform beam of 200N is supported horizontally as shown. If the breaking tension of the rope is 400N, how far can the man of weight 400 N walk from point A on the beam as shown in Fig. 2.18?

Solution: Let breaking point be at a distance d from the pivot A. The force diagram of the situation is given in Fig 2.19. By applying 2nd condition of equilibrium about point A

$$\Sigma\tau = 0$$

$$400\text{ N} \times 6\text{ m} - 400\text{ N} \times d - 200\text{ N} \times 3\text{ m} = 0$$

or $$400\text{ N} \times d = 2400\text{ Nm} - 600\text{ Nm} = 1800\text{ Nm}$$

$$d = 4.5\text{ m}$$

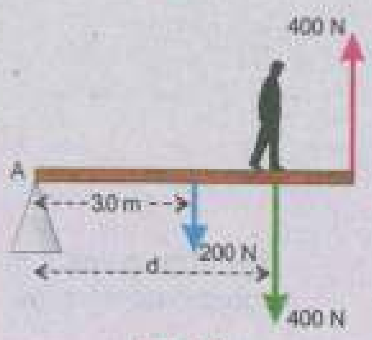


Fig. 2.19

Example 2.9: A boy weighing 300 N is standing at the edge of a uniform diving board 4.0m in length. The weight of the board is 200 N. (Fig. 2.20 a). Find the forces exerted by pedestals on the board.

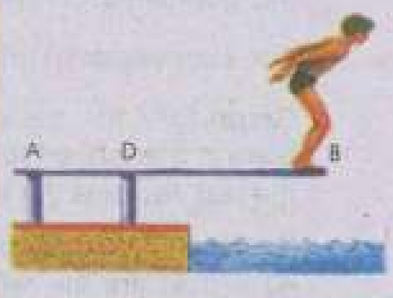


Fig. 2.20(a)

Solution: We isolate the diving board which is in equilibrium under the action of forces shown in the force diagram (Fig. 2.20 b). Note that the weight 200 N of the uniform diving board is shown to act at point C, the centre of gravity which is taken as the mid-point of the board. R_1 and R_2 are the reaction forces exerted by the pedestals on the board. A little consideration will show that R_1 is in the wrong direction, because the board must be actually pressed down in order to keep it in equilibrium. We shall see that this assumption will be automatically corrected by calculations.

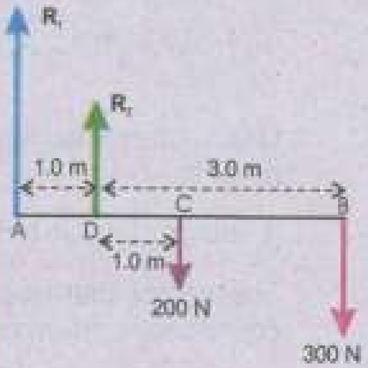


Fig. 2.20(b)

Let us now apply conditions of equilibrium

$\Sigma F_x = 0$ (No x-directed forces)

$\Sigma F_y = 0$ $R_1 + R_2 - 300 - 200 = 0$

$R_1 + R_2 = 500$ N ... (i)

$\Sigma\tau = 0$ (pivot at point D)

$$-R_1 \times AD - 300\text{ N} \times DB - 200\text{ N} \times DC = 0$$

$$-R_1 \times 1\text{m} - 300\text{ N} \times 3\text{m} - 200\text{ N} \times 1\text{m} = 0$$

$$R_1 = -1100\text{ N} = -1.1\text{ kN}$$

Substituting the value of R_1 in Eq. (i). we have

$$-1100 + R_2 = 500$$

$$R_2 = 1600\text{ N} = 1.6\text{ kN}$$

The negative sign of R_1 shows that it is directed downward. Thus the result has corrected the mistake of our initial assumption.

SUMMARY

- The arrangement of mutually perpendicular axes is called rectangular or Cartesian coordinate system.
- A scalar is a quantity that has magnitude only, whereas a vector is a quantity that has both direction and magnitude.
- The sum vector of two or more vectors is called resultant vector.
- Graphically the vectors are added by drawing them to a common scale and placing them head to tail, the vector connecting the tail of the first to the head of the last vector is the resultant vector.
- Vector addition can be carried out by using rectangular components of vectors. If A_x and A_y are the rectangular components of **A** and B_x and B_y are that of vector **B**, then the sum **R** = **A** + **B** is given by

  $$R_x = A_x + B_x \quad , \quad R_y = A_y + B_y$$

  where $R = \sqrt{R_x^2 + R_y^2}$ and direction $\theta = \tan^{-1}\frac{R_y}{R_x}$
- Unit vectors describe directions in space. A unit vector has a magnitude of 1 with no units.
- A vector of magnitude zero without any specific direction is called null vector.
- The vector that describes the location of a particle with respect to the origin of coordinate system is known as position vector.
- The scalar product of two vectors **A** and **B** is a scalar quantity, defined as :

  $$\mathbf{A}.\mathbf{B} = AB\cos\theta$$

- The vector product of two vectors **A** and **B** is another vector **C** whose magnitude is given by : $C = AB\sin\theta$

  Its direction is perpendicular to the plane of the two vectors being multiplied, as given by the right hand rule.

- A body is said to be in equilibrium under the action of several forces if the body has zero translational acceleration and no angular acceleration.
- For a body to be in translational equilibrium the vector sum of all the forces acting on the body must be zero.
- The torque is defined as the product of the force and the moment arm.
- The moment arm is the perpendicular distance from the axis of rotation to the direction of line of action of the force.
- For a body to be in rotational equilibrium, the sum of torques on the body about any axis must be equal to zero.

QUESTIONS

2.1 Define the terms (i) unit vector (ii) Position vector and (iii) Components of a vector.

2.2 The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

2.3 Vector **A** lies in the xy plane. For what orientation will both of its rectangular components be negative ? For what orientation will its components have opposite signs?

2.4 If one of the rectangular components of a vector is not zero, can its magnitude be zero ? Explain.

2.5 Can a vector have a component greater than the vector's magnitude?

2.6 Can the magnitude of a vector have a negative value?

2.7 If **A** + **B** = **0**, What can you say about the components of the two vectors?

2.8 Under what circumstances would a vector have components that are equal in magnitude?

2.9 Is it possible to add a vector quantity to a scalar quantity? Explain.

2.10 Can you add zero to a null vector?

2.11 Two vectors have unequal magnitudes. Can their sum be zero? Explain.

2.12 Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of the same length.

2.13 How would the two vectors of the same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude?

2.14 The two vectors to be combined have magnitudes 60 N and 35 N. Pick the correct answer from those given below and tell why is it the only one of the three that is correct.

i) 100 N ii) 70 N iii) 20 N

2.15 Suppose the sides of a closed polygon represent vector arranged head to tail. What is the sum of these vectors?

2.16 Identify the correct answer.

i) Two ships X and Y are travelling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be

(A) East (B) West (C) south-east (D) south-west

ii) A horizontal force F is applied to a small object P of mass *m* at rest on a smooth plane inclined at an angle θ to the horizontal as shown in Fig. 2.22. The magnitude of the resultant force acting up and along the surface of the plane, on the object is

a) $F\cos\theta - mg\sin\theta$

b) $F\sin\theta - mg\cos\theta$

c) $F\cos\theta + mg\cos\theta$

d) $F\sin\theta + mg\sin\theta$

e) $mg\tan\theta$

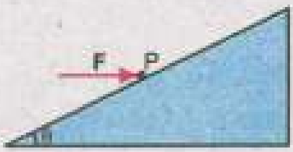


Fig. 2.21

2.17 If all the components of the vectors, $\mathbf{A}_1$ and $\mathbf{A}_2$ were reversed, how would this alter $\mathbf{A}_1 \times \mathbf{A}_2$?

2.18 Name the three different conditions that could make $\mathbf{A}_1 \times \mathbf{A}_2 = 0$.

2.19 Identify true or false statements and explain the reason.

a) A body in equilibrium implies that it is not moving nor rotating.

b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

2.20 A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the strings will be minimum.

2.21 Can a body rotate about its centre of gravity under the action of its weight?

NUMERICAL PROBLEMS

2.1. Suppose, in a rectangular coordinate system, a vector **A** has its tail at the point P (-2, -3) and its tip at Q (3,9). Determine the distance between these two points.

(Ans: 13 Units)

2.2. A certain corner of a room is selected as the origin of a rectangular coordinate system. If an insect is sitting on an adjacent wall at a point having coordinates (2,1), where the units are in metres, what is the distance of the insect from this corner of the room?

(Ans: 2.2m)

2.3. What is the unit vector in the direction of the vector $\mathbf{A}=4\hat{i}+3\hat{j}$?

(Ans: $\frac{(4\hat{i}+3\hat{j})}{5}$)

2.4. Two particles are located at $\mathbf{r}_1=3\hat{i}+7\hat{j}$ and $\mathbf{r}_2=-2\hat{i}+3\hat{j}$ respectively. Find both the magnitude of the vector $(\mathbf{r}_2-\mathbf{r}_1)$ and its orientation with respect to the x-axis.

[Ans: 6.4,219°]

2.5. If a vector **B** is added to vector **A**, the result is $6\hat{i}+\hat{j}$. If **B** is subtracted from **A**, the result is $-4\hat{i}+7\hat{j}$. What is the magnitude of vector **A**?

(Ans: 4.1)

2.6. Given that $\mathbf{A}=2\hat{i}+3\hat{j}$ and $\mathbf{B}=3\hat{i}-4\hat{j}$, find the magnitude and angle of (a) $\mathbf{C}=\mathbf{A}+\mathbf{B}$, and (b) $\mathbf{D}=3\mathbf{A}-2\mathbf{B}$.

(Ans: 5.1, 349°; 17,90°)

2.7. Find the angle between the two vectors, $\mathbf{A}=5\hat{i}+\hat{j}$ and $\mathbf{B}=2\hat{i}+4\hat{j}$.

(Ans: 52°)

2.8. Find the work done when the point of application of the force $3\hat{i}+2\hat{j}$ moves in a straight line from the point (2,-1) to the point (6,4).

(Ans: 22 units)

2.9. Show that the three vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}-3\hat{j}+\hat{k}$ and $4\hat{i}+\hat{j}-5\hat{k}$ are mutually perpendicular.

2.10. Given that $\mathbf{A} = \hat{i}-2\hat{j}+3\hat{k}$ and $\mathbf{B}=3\hat{i}-4\hat{k}$, find the projection of **A** on **B**.

(Ans: $-\frac{9}{5}$)

2.11. Vectors **A**, **B** and **C** are 4 units north, 3 units west and 8 units east, respectively. Describe carefully (a) **A** x **B** (b) **A** x **C** (c) **B** x **C**

[Ans: (a) 12 units vertically up (b) 32 units vertically down (c) Zero]

2.12. The torque or turning effect of force about a given point is given by **r** x **F** where **r** is the vector from the given point to the point of application of **F**. Consider a force $\mathbf{F} = -3\hat{i}+\hat{j}+5\hat{k}$ (newton) acting on the point $7\hat{i}+3\hat{j}+\hat{k}$ (m). What is the torque in N m about the origin?

[Ans: $14\hat{i}-38\hat{j}+16\hat{k}$ Nm]

2.13. The line of action of force, $\mathbf{F} = \hat{i}-2\hat{j}$, passes through a point whose position vector is $(-\hat{j}+\hat{k})$. Find (a) the moment of **F** about the origin, (b) the moment of **F** about the point of which the position vector is $\hat{i}+\hat{k}$.

[Ans: (a) $2\hat{i}+\hat{j}+\hat{k}$, (b) $3\hat{k}$]

2.14. The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors

(Ans: 30°)

2.15. A load of 10.0 N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.

[Ans: 19.3N]

Chapter 3

MOTION AND FORCE

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand displacement from its definition and illustration.
2. Understand velocity, average velocity and instantaneous velocity.
3. Understand acceleration, average acceleration and instantaneous acceleration.
4. Understand the significance of area under velocity-time graph.
5. Recall and use equations, which represent uniformly accelerated motion in a straight line including falling in a uniform gravitational field without air resistance.
6. Recall Newton's Laws of motion.
7. Describe Newton's second law of motion as rate of change of momentum.
8. Define impulse as a product of impulsive force and time.
9. Describe law of conservation of momentum.
10. Use the law of conservation of momentum in simple applications including elastic collisions between two bodies in one dimension.
11. Describe the force produced due to flow of water.
12. Understand the process of rocket propulsion (simple treatment).
13. Understand projectile motion in a non-resistive medium.
14. Derive time of flight, maximum height and horizontal range of projectile motion.
15. Appreciate the motion of ballistic missiles as projectile motion.

We live in a universe of continual motion. In every piece of matter, the atoms are in a state of never ending motion. We move around the Earth's surface, while the Earth moves in its orbit around the Sun. The Sun and the stars, too, are in motion. Everything in the vastness of space is in a state of perpetual motion.

Every physical process involves motion of some sort. Because of its importance in the physical world around us, it is logical that we should give due attention to the study of motion.

We already know that motion and rest are relative. Here, in this chapter, we shall discuss other related topics in some more details.

3.1 DISPLACEMENT

Whenever a body moves from one position to another, the change in its position is called displacement. The displacement can be represented as a vector that describes how far and in what direction the body has been displaced from its original position. The tail of the displacement vector is located at the position where the displacement started, and its tip or arrowhead is located at the final position where the displacement ended. For example, if a body is moving along a curve as shown in Fig. 3.1 with A as its initial position and B as its final position then the displacement **d** of the body is represented by **AB**. Note that although the body is moving along a curve, the displacement is different from the path of motion.

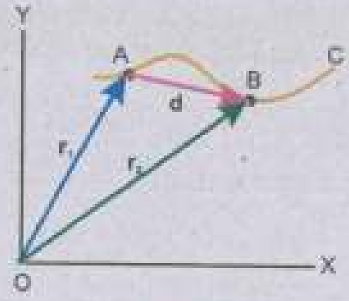


Fig.3.1

If $\mathbf{r}_1$ is the position vector of A and $\mathbf{r}_2$ that of point B then by head and tail rule it can be seen from the figure that

$$\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$$

> **The displacement is thus a change in the position of body from its initial position to its final position.**

Its magnitude is the straight line distance between the initial position and the final position of the body.

When a body moves along a straight line, the displacement coincides with the path of motion as shown in Fig. 3.2. (a)

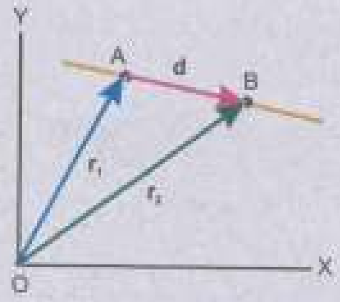


Fig.3.2(a)

3.2 VELOCITY

We have studied in school physics that time rate of change of displacement is known as velocity. Its direction is along the direction of displacement. So if **d** is the total displacement of the body in time t, then its average velocity during the interval t is defined as

$$\mathbf{v}_{av} = \frac{\mathbf{d}}{t} \qquad \text{..........} \qquad (3.1)$$

Average velocity does not tell us about the motion between A and B. The path may be straight or curved and the motion may be steady or variable. For example if a squash ball comes back to its starting point after bouncing off the wall several times, its total displacement is zero and so also is its average velocity.

In such cases the motion is described by the instantaneous velocity.

In order to understand the concept of instantaneous velocity, consider a body moving along a path ABC in xy plane. At any time t, let the body be at point A Fig.3.2(b).Its position is given by position vector $\mathbf{r}_1$. After a short time interval Δt following the instant t, the body reaches the point B which is described by the position vector $\mathbf{r}_2$. The displacement of the body during this short time interval is given by

$$\Delta \mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$$

The notation Δ (delta) is used to represent a very small change.

The instantaneous velocity at a point A, can be found by making Δt smaller and smaller. In this case $\Delta \mathbf{d}$ will also become smaller and point B will approach A. If we continue this process, letting B approach A, thus, allowing Δt and $\Delta \mathbf{d}$ to decrease but never disappear completely, the ratio $\Delta \mathbf{d}/\Delta t$ approaches a definite limiting value which is the instantaneous velocity. Although Δt and $\Delta \mathbf{d}$ become extremely small in this process, yet their ratio is not necessarily a small quantity. Moreover, while decreasing the displacement vector, $\Delta \mathbf{d}$ approaches a limiting direction along the tangent at A. Therefore,

The instantaneous velocity is defined as the limiting value of $\Delta d/\Delta t$ as the time interval Δt, following the time t, approaches zero.

For Your Information

Typical Speeds

Speed, ms^{-1}	Motion
300 000 000	Light, radio waves, X-rays, microwaves (in vacuum)
210 000	Earth-Sun travel around the galaxy
* 29 600	Earth around the Sun
1 000	Moon around the Earth
980	SR-71 reconnaissance jet
333	Sound (in air)
267	Commercial jet airliner
62	Commercial automobile (max.)
37	Falcon in a dive
29	Running cheetah
10	100-metres dash (max.)
9	Porpoise swimming
5	Flying bee
4	Human running
2	Human swimming
0.01	Walking ant

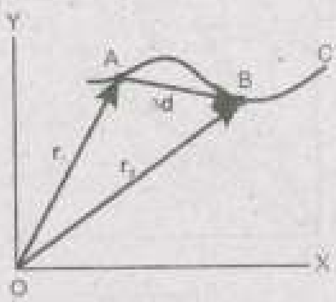


Fig.3.2(b)

Using the mathematical language, the definition of instantaneous velocity $\mathbf{v}_{ins}$ is expressed as

$$\mathbf{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{d}}{\Delta t} \quad \text{..........} \quad (3.2)$$

read as limiting value of $\Delta \mathbf{d}/\Delta t$ as Δt approaches zero.

If the instantaneous velocity does not change, the body is said to be moving with uniform velocity.

3.3 ACCELERATION

If the velocity of an object changes, it is said to be moving with an acceleration.

The time rate of change of velocity of a body is called acceleration.

As velocity is a vector so any change in velocity may be due to change in its magnitude or a change in its direction or both.

Consider a body whose velocity $\mathbf{v}_1$ at any instant t changes to $\mathbf{v}_2$ in further small time interval Δt. The two velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ and the change in velocity, $\mathbf{v}_2 - \mathbf{v}_1 = \Delta \mathbf{v}$, are represented in Fig. 3.3. The average acceleration $\mathbf{a}_{av}$ during time interval Δt is given by

Fig.3.3

$$\mathbf{a}_{av} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{\Delta t} = \frac{\Delta \mathbf{v}}{\Delta t} \quad \text{..........} \quad (3.3)$$

As $\mathbf{a}_{av}$ is the difference of two vectors divided by a scalar Δt, $\mathbf{a}_{av}$ must also be a vector. Its direction is the same as that of $\Delta \mathbf{v}$. Acceleration of a body at a particular instant is known as instantaneous acceleration and it is the value obtained from the average acceleration as Δt is made smaller and smaller till it approaches zero. Mathematically, it is expressed as

$$\text{Instantaneous acceleration} = \mathbf{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} \quad \text{..........} \quad (3.4)$$

If the velocity of a body is increasing, its acceleration is positive but if the velocity is decreasing the acceleration is negative. If the velocity of the body changes by equal amount in equal intervals of time, the body is said to have uniform acceleration. For a body moving with uniform acceleration, its average acceleration is equal to instantaneous acceleration.

3.4 VELOCITY-TIME GRAPH

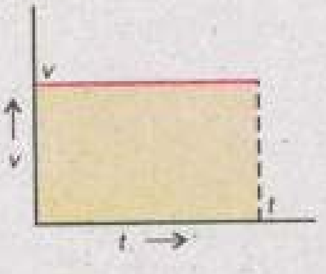


Fig.3.4

Graphs may be used to illustrate the variation of velocity of an object with time. Such graphs are called velocity-time graphs. The velocity.time graphs of an object making three different journeys along a straight road are shown in figures 3.4 to 3.6. When the velocity of the car is constant, its velocity-time graph is a horizontal straight line (Fig 3.4). When the car moves with constant acceleration, the velocity-time graph is a straight line which rises the same height for equal intervals of time (Fig 3.5).

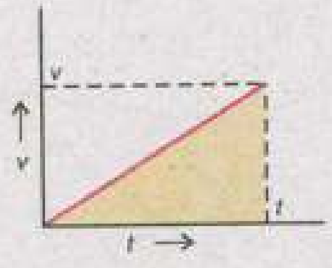


Fig.3.5

The average acceleration of the car during the interval *t* is given by the slope of its velocity-time graph.

When the car moves with increasing acceleration, the velocity-time graph is a curve (Fig 3.6). The point A on the graph corresponds to time *t*. The magnitude of the instantaneous acceleration at this instant is numerically equal to the slope of the tangent at the point A on the velocity-time graph of the object as shown in Fig 3.6.

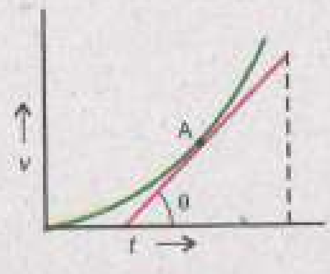
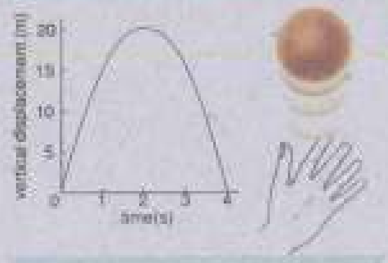


Fig.3.6

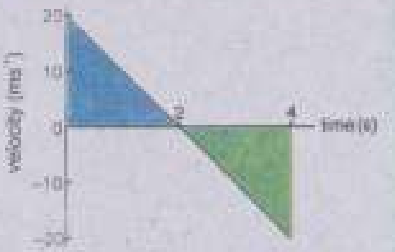
The distance moved by an object can also be determined by using its velocity-time graph. For example, Fig 3.4 shows that the object moves at constant velocity **v** for time *t*. The distance covered by the object given by Eq. 3.1 is $v \times t$. This distance can also be found by calculating the area under the velocity-time graph. This area is shown shaded in Fig 3.4 and is equal to $v \times t$. We now give another example shown in Fig 3.5. Here the velocity of the object increases uniformly from 0 to **v** in time *t*. The magnitude of its average velocity is given by

$$v_{av} = \frac{0 + v}{2} = \frac{1}{2}v$$

Do You Know?

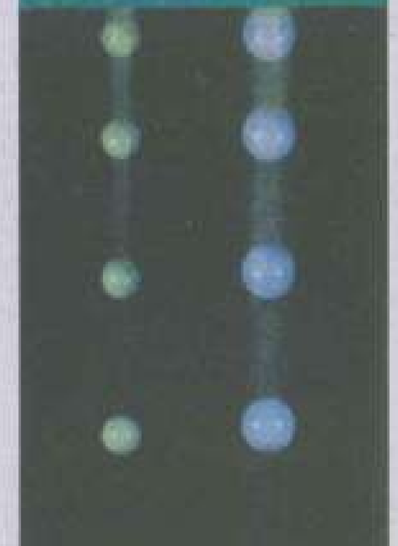


How the displacement of a vertically thrown ball varies with time?



How the velocity of a vertically thrown ball varies with time? Velocity is upwards positive.

Do You Know?



At the surface of the Earth, in situations where air friction is negligible, objects fall with the same acceleration regardless of their weights.

3.5 REVIEW OF EQUATIONS OF UNIFORMLY ACCELERATED MOTION

In school physics we have studied some useful equations for objects moving at constant acceleration.

Suppose an object is moving with uniform acceleration a along a straight line. If its initial velocity is v_i and final velocity after a time interval t is v_f. Let the distance covered during this interval be S then we have

$$v_f = v_i + at \qquad \text{..........} \qquad (3.5)$$

$$S = \frac{(v_f + v_i)}{2} \times t \qquad \text{..........} \qquad (3.6)$$

$$S = v_i t + \frac{1}{2} at^2 \qquad \text{..........} \qquad (3.7)$$

$$v_f^2 = v_i^2 + 2aS \qquad \text{..........} \qquad (3.8)$$

These equations are useful only for linear motion with uniform acceleration. When the object moves along a straight line, the direction of motion does not change. In this case all the vectors can be manipulated like scalars. In such problems, the direction of initial velocity is taken as positive. A negative sign is assigned to quantities where direction is opposite to that of initial velocity.

In the absence of air resistance, all objects in free fall near the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration, known as acceleration due to gravity, is denoted by the letter **g** and its average value near the Earth surface is taken as $9.8\ ms^{-2}$ in the downward direction.

The equations for uniformly accelerated motion can also be applied to free fall motion of the objects by replacing a by g.

3.6 NEWTON'S LAWS OF MOTION

Newton's laws are empirical laws, deduced from experiments. They were clearly stated for the first time by Sir Isaac Newton, who published them in 1687 in his famous book called "Principia". Newton's laws are adequate for speeds that are low compared with the speed of light.

Distance covered = average velocity x time = $\frac{1}{2} v \times t$

Now we calculate the area under velocity-time graph which is equal to the area of the triangle shaded in Fig 3.5. Its value is equal to 1/2 base x height = 1/2 v x t. Considering the above two examples it is a general conclusion that

The area between the velocity-time graph and the time axis is numerically equal to the distance covered by the object.

Example 3.1: The velocity-time graph of a car moving on a straight road is shown in Fig 3.7. Describe the motion of the car and find the distance covered.

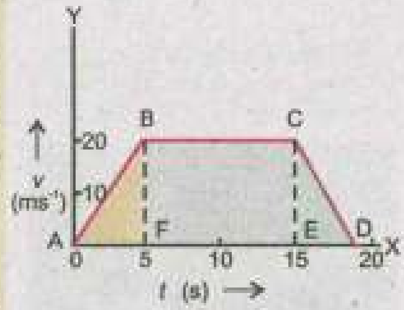


Fig.3.7

Solution: The graph tells us that the car starts from rest, and its velocity increases uniformly to 20 ms^{-1} in 5 seconds. Its average acceleration is given by

$$a = \frac{\Delta v}{\Delta t} = \frac{20\ \text{ms}^{-1}}{5\ \text{s}} = 4\ \text{ms}^{-2}$$

The graph further tells us that the velocity of the car remains constant from 5th to 15th second and it then decreases uniformly to zero from 15th to 19th seconds. The acceleration of the car during last 4 seconds is

$$a = \frac{\Delta v}{\Delta t} = \frac{-20\ \text{ms}^{-1}}{4\ \text{s}} = -5\ \text{ms}^{-2}$$

The negative sign indicates that the velocity of the car decreases during these 4 seconds.

The distance covered by the car is equal to the area between the velocity-time graph and the time-axis. Thus

Distance travelled = Area of ΔABF + Area of rectangle BCEF + Area of ΔCDE

$$= \frac{1}{2} \times 20\ \text{ms}^{-1} \times 5\ \text{s} + 20\ \text{ms}^{-1} \times 10\ \text{s} + \frac{1}{2} \times 20\ \text{ms}^{-1} \times 4\ \text{s}$$

$$= 50\ \text{m} + 200\ \text{m} + 40\ \text{m} = 290\ \text{m}$$

For very fast moving objects, such as atomic particles in an accelerator, relativistic mechanics developed by Albert Einstein is applicable.

You have already studied these laws in your secondary school Physics. However a summarized review is given below.

An unappreciated anticipation.

No body begins to move or comes to rest of itself.

ABU ALI SENA, (980-1037)

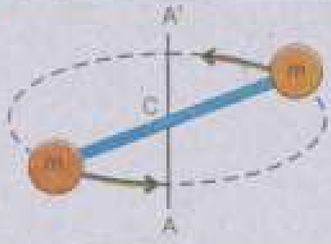
Newton's First Law of Motion

A body at rest will remain at rest, and a body moving with uniform velocity will continue to do so, unless acted upon by some unbalanced external force. This is also known as law of inertia. The property of an object tending to maintain the state of rest or state of uniform motion is referred to as the object's inertia. The more inertia, the stronger is this tendency in the presence of a force. Thus,

The mass of the object is a quantitative measure of its inertia.

The frame of reference in which Newton's first law of motion holds, is known as inertial frame of reference. A frame of reference stationed on Earth is approximately an inertial frame of reference.

For Your Information



A measurement of mass independent of gravity. The unknown mass m and a calibrated mass m_1 are mounted on a light weight rod. If the masses are equal, the rod will rotate without wobble about its centre.

Newton's Second Law of Motion

A force applied on a body produces acceleration in its own direction. The acceleration produced varies directly with the applied force and inversely with the mass of the body. Mathematically, it is expressed as

$$F = ma \quad \text{..........} \quad (3.9)$$

Newton's Third Law of Motion

Action and reaction are equal and opposite. For example, whenever an interaction occurs between two objects, each object exerts the same force on the other, but in the opposite direction and for the same length of time. Each force in action-reaction pair acts only on one of the two bodies, the action and reaction forces never act on the same body.

Point to Ponder

A car accelerates along a road. Which force actually moves the car?

3.7 MOMENTUM

We are aware of the fact that moving object possesses a quality by virtue of which it exerts a force on anything that tries to stop it. The faster the object is travelling, the harder is to stop it. Similarly, if two objects move with the same velocity, then it is more difficult to stop the massive of the two.

This quality of the moving body was called the quantity of motion of the body, by Newton. This term is now called linear momentum of the body and is defined by the relation.

Linear momentum = $\mathbf{p} = m\,\mathbf{v}$ .... (3.10)

In this expression $\mathbf{v}$ is the velocity of the mass m. Linear momentum is, therefore, a vector quantity and has the direction of velocity.

The SI unit of momentum is kilogram metre per second (kg m s^{-1}). It can also be expressed as newton second (N s).

Interesting Information

Throwing a package onto shore from a boat that was previously at rest causes the boat to move out-ward from shore (Newton's third law).

Momentum and Newton's Second Law of Motion

Consider a body of mass m moving with an initial velocity $\mathbf{v}_i$. Suppose an external force $\mathbf{F}$ acts upon it for time t after which velocity becomes $\mathbf{v}_f$. The acceleration $\mathbf{a}$ produced by this force is given by

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

By Newton's second law, the acceleration is given as

$$\mathbf{a} = \frac{\mathbf{F}}{m}$$

Equating the two expressions of acceleration, we have

$$\frac{\mathbf{F}}{m} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

or $\mathbf{F} \times t = m\,\mathbf{v}_f - m\,\mathbf{v}_i$ .... (3.11)

where $m\mathbf{v}_i$ is the initial momentum and $m\mathbf{v}_f$ is the final momentum of the body

Point to Ponder

Which will be more effective in knocking a bear down.

i. a rubber bullet or
ii. a lead bullet of the same momentum

The equation 3.11 shows that change in momentum is equal to the product of force and the time for which force is applied. This form of the second law is more general than the form **F** = *m***a**, because it can easily be extended to account for changes as the body accelerates when its mass also changes. For example, as a rocket accelerates, it loses mass because its fuel is burnt and ejected to provide greater thrust.

From Eq. 3.11.

$$\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{t}$$

Thus, second law of motion can also be stated in terms of momentum as follows

Time rate of change of momentum of a body equals the applied force.

Impulse

Sometimes we wish to apply the concept of momentum to cases where the applied force is not constant, it acts for very short time. For example, when a bat hits a cricket ball, the force certainly varies from instant to instant during the collision. In such cases, it is more convenient to deal with the product of force and time (**F** x *t*) instead of either quantity alone. The quantity **F** x *t* is called the impulse of the force, where **F** can be regarded as the average force that acts during the time *t*. From Eq. 3.11

$$\text{Impulse} = \mathbf{F} \times t = m\,\mathbf{v}_f - m\,\mathbf{v}_i \quad \text{..........} \quad (3.12)$$

Example 3.2: A 1500 kg car has its velocity reduced from 20 ms⁻¹ to 15 ms⁻¹ in 3.0 s. How large was the average retarding force?

Solution: Using the Eq 3.11

$$F \times t = m\,v_f - m\,v_i$$

$$F \times 3.0\ \text{s} = 1500\ \text{kg} \times 15\ \text{ms}^{-1} - 1500\ \text{kg} \times 20\ \text{ms}^{-1}$$

$$\text{or } F = -2500\ \text{kg ms}^{-2} = -2500\ \text{N} \quad -2.5\ \text{kN}$$

The negative sign indicates that the force is retarding one.

Point to Ponder

Which hurt you in the above situations (a) or (b) and think why?

Point to Ponder

Does a moving object have impulse?

Do You Know?

Your hair acts like a crumple zone on your skull. A force of 5N might be enough to fracture your naked skull (cranium), but with a covering of skin and hair, a force of 50 N would be needed.

Law of Conservation of Momentum

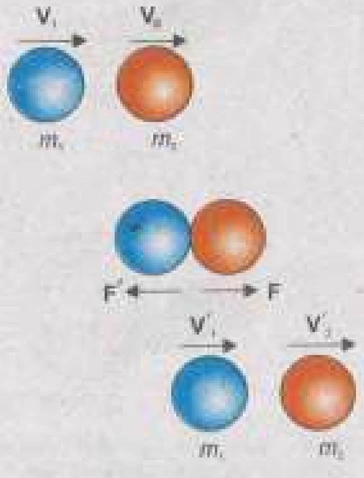


Fig. 3.8

Let us consider an isolated system. It is a system on which no external agency exerts any force. For example, the molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion but, being enclosed by glass vessel, no external agency can exert a force on them.

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Both the balls collide and after collision, ball of mass m_1 moves with velocity $\mathbf{v}'_1$ and m_2 moves with velocity $\mathbf{v}'_2$ in the same direction as shown in Fig 3.8.

To find the change in momentum of mass m_1, using Eq 3.11 we have,

$$\mathbf{F}' \times t = m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1$$

Similarly for the ball of mass m_2, we have

$$\mathbf{F} \times t = m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2$$

Adding these two expressions, we get

$$(\mathbf{F} + \mathbf{F}')t = (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2)$$

Since the action force $\mathbf{F}$ is equal and opposite to the reaction force $\mathbf{F}'$, we have $\mathbf{F}' = -\mathbf{F}$, so the left hand side of the equation is zero. Hence,

$$0 = (m_1 \mathbf{v}'_1 - m_1 \mathbf{v}_1) + (m_2 \mathbf{v}'_2 - m_2 \mathbf{v}_2)$$

In other words, change in momentum of 1st ball + change in momentum of the 2nd ball = 0

Or $(m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2) = (m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2)$ ...... (3.13)

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

For such a group of objects, if one object within the group experiences a force, there must exist an equal but

Point to Ponder

What is the effect on the speed of a fighter plane chasing another when it opens fire? What happens to the speed of pursued plane when it returns the fire?

opposite reaction force on some other object in the same group. As a result, the change in momentum of the group of objects as a whole is always zero. This can be expressed in the form of law of conservation of momentum which states that

The total linear momentum of an isolated system remains constant.

In applying the conservation law, we must notice that the momentum of a body is a vector quantity.

Example 3.3: Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other with velocities of 6.0 ms^{-1} and 4 ms^{-1} respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 ms^{-1}?

Solution: As both the balls are moving towards one another, so their velocities are of opposite sign. Let us suppose that the direction of motion of 2 kg ball is positive and that of the 3 kg is negative.

The momentum of the system before collision = $m_1 v_1 + m_2 v_2$

$= 2\ \text{kg} \times 6\ \text{ms}^{-1} + 3\ \text{kg} \times (-4\ \text{ms}^{-1}) = 12\ \text{kgms}^{-1} - 12\ \text{kg m s}^{-1} = 0$

Momentum of the system after collision = $m_1 v'_1 + m_2 v'_2$

$= 2\ \text{kg} \times v'_1 + 3\ \text{kg} \times (-3)\text{ms}^{-1}$

From the law of conservation of momentum

$$\begin{bmatrix}\text{Momentum of the system} \\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Momentum of the system} \\ \text{after collision}\end{bmatrix}$$

$$0 = 2\ \text{kg} \times v'_1 - 9\ \text{kg m s}^{-1}$$

$$2\ \text{kg} \times v'_1 = 9\ \text{kg m s}^{-1}$$

$$v'_1 = 4.5\ \text{m s}^{-1}$$

Do you wear seat belts?

When a moving car stops quickly, the passengers move forward towards the windshield. Seat belts change the forces of motion and prevent the passengers from moving. Thus the chance of injury is greatly reduced.

Do You Know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

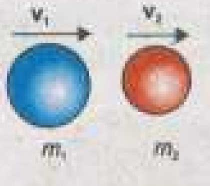
3.8 ELASTIC AND INELASTIC COLLISIONS

When two tennis balls collide then, after collision, they will rebound with velocities less than the velocities before the impact. During this process, a portion of K.E is lost, partly due to friction as the molecules in the ball move past one another when the balls distort and partly due to its change into heat and sound energies.

A collision in which the K.E of the system is not conserved, is called the inelastic collision.

Under certain special conditions no kinetic energy is lost in the collision.

In the ideal case when no K.E is lost, the collision is said to be perfectly elastic.

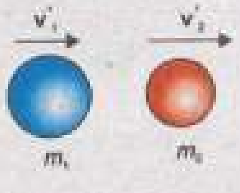


Before collision

For example, when a hard ball is dropped onto a marble floor, it rebounds to very nearly the initial height. It looses negligible amount of energy in the collision with the floor.

It is to be noted that momentum and total energy are conserved in all types of collisions. However, the K.E. is conserved only in elastic collisions.

Elastic Collision in One Dimension



After collision

Fig. 3.9

Consider two smooth, non-rotating balls of masses m_1 and m_2, moving initially with velocities v_1 and v_2 respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after the collision be v'_1 and v'_2 respectively, as shown in Fig. 3.9.

We take the positive direction of the velocity and momentum to the right. By applying the law of conservation of momentum we have

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

$$m_1(v_1 - v'_1) = m_2(v'_2 - v_2) \quad \text{..........} \quad (3.14)$$

As the collision is elastic, so the K.E is also conserved. From the conservation of K.E we have

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

or $$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

or $$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2) \quad .... \quad (3.15)$$

Dividing equation 3.15 by 3.14

$$(v_1 + v_1') = (v_2' + v_2) \quad .... \quad (3.16)$$

or $$(v_1 - v_2) = (v_2' - v_1') = -(v_1' - v_2')$$

We note that, before collision $(v_1 - v_2)$ is the velocity of first ball relative to the second ball. Similarly $(v_1' - v_2')$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

In equations 3.14 and 3.16, m_1, m_2, v_1 and v_2 are known quantities. We solve these equations to find the values of v_1' and v_2', which are unknown. The results are

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2 \quad .... \quad (3.17)$$

$$v_2' = \frac{2m_1}{m_1 + m_2}v_1 + \frac{m_2 - m_1}{m_1 + m_2}v_2 \quad .... \quad (3.18)$$

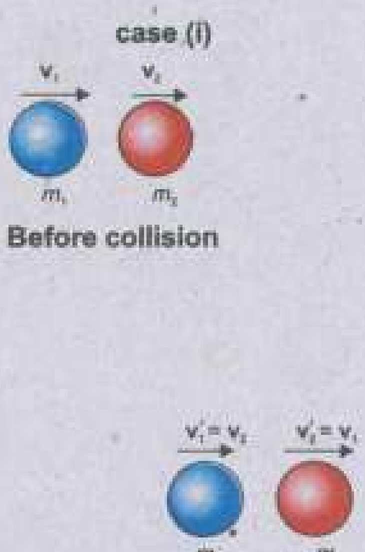


Fig. 3.10

There are some cases of special interest, which are discussed below:

(i) When $\quad m_1 = m_2$

From equations 3.17 and 3.18 we find that

$$v_1' = v_2$$

and $\quad v_2' = v_1 \quad$ as shown in Fig 3.10

(ii) When $\quad m_1 = m_2$ **and** $v_2 = 0$

In this case the mass m_2 be at rest, then $v_2 = 0$ the equations 3.17 and 3.18 give case (ii)

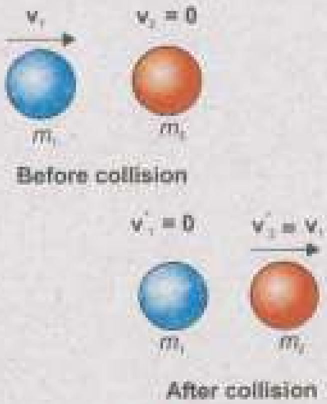


Fig. 3.11

case (iii)

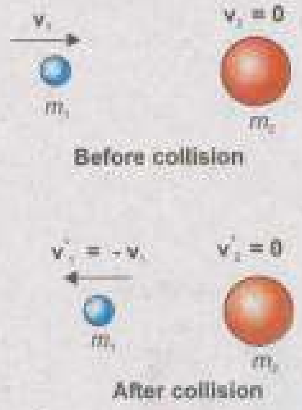


Fig. 3.12

case (iv)

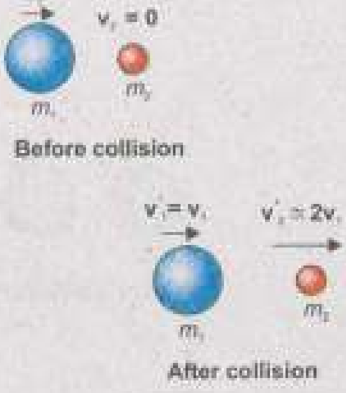


Fig. 3.13

$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad ; \quad v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

When $m_1 = m_2$ then ball of mass m_1 after collision will come to a stop and m_2 will take off with the velocity that m_1 originally has, as shown in Fig 3.11. Thus when a billiard ball m_1, moving on a table collides with exactly similar ball m_2 at rest, the ball m_1 stops while m_2 begins to move with the same velocity, with which m_1 was moving initially.

(iii) When a light body collides with a massive body at rest

In this case initial velocity $v_2 = 0$ and $m_2 >> m_1$. Under these conditions m_1 can be neglected as compared to m_2. From equations 3.17 and 3.18 we have $v'_1 = -v_1$ and $v'_2 = 0$

The result is shown in Fig 3.12. This means that m_1 will bounce back with the same velocity while m_2 will remain stationary. This fact is made used of by the squash player.

(iv) When a massive body collides with light stationary body

In this case $m_1 >> m_2$ and $v_2 = 0$ so m_2 can be neglected in equations 3.17 and 3.18. This gives $v'_1 \simeq v_1$ and $v'_2 \simeq 2\,v_1$. Thus after the collision, there is practically no change in the velocity of the massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body, as shown in Fig. 3.13.

Example 3.4: A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 ms^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic what would be the velocity of the two balls after the collision?

Solution:

$m_1 = 70$ g $\quad v_1 = 9$ ms^{-1} $\quad v_2 = 0$

$m_2 = 140$ g $\quad v'_1 = ?$ $\quad v'_2 = ?$

We know that $\quad v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1$

$$= \frac{70\,g - 140\,g}{70\,g + 140\,g} \times 9\ ms^{-1} = -3\ ms^{-1}$$

$$v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

$$= \frac{2 \times 70\,g}{70\,g + 140\,g} \times 9\ ms^{-1} = 6\ ms^{-1}$$

Example 3.5: A 100 g golf ball is moving to the right with a velocity of 20 ms^{-1}. It makes a head on collision with an 8 kg steel ball, initially at rest. Compute velocities of the balls after collision.

Solution: We know that

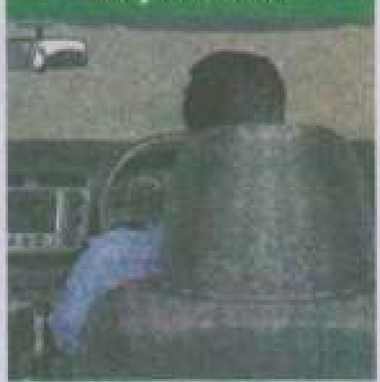
$$v'_1 = \frac{m_1 - m_2}{m_1 + m_2} v_1 \quad \text{and} \quad v'_2 = \frac{2m_1}{m_1 + m_2} v_1$$

Hence

$$v'_1 = \frac{0.1\,kg - 8\,kg}{0.1\,kg + 8\,kg} \times 20\ ms^{-1} = -19.5\ ms^{-1}$$

$$v'_2 = \frac{2 \times 0.1\,kg}{0.1\,kg + 8\,kg} \times 20\ ms^{-1} = 0.5\ ms^{-1}$$

Do you know?



If another car crashes into back of yours, the head-rest of the car seat can save you from serious neck injury. It helps to accelerate your head forward with the same rate as the rest of your body.

3.9 FORCE DUE TO WATER FLOW

When water from a horizontal pipe strikes a wall normally, a force is exerted on the wall. Suppose the water strikes the wall normally with velocity *v* and comes to rest on striking the wall, the change in velocity is then $0 - v = -v$. From second law, the force equals the momentum change per second of water. If mass *m* of the water strikes the wall in time *t* then force *F* on the water is

$$F = -\frac{m}{t} v = -\text{mass per second} \times \text{change in velocity} \quad \text{.... (3.19)}$$

Point to Ponder

In thrill machine rides at amusement parks, there can be an acceleration of 3g or more. But without head rests, acceleration like this would not be safe. Think why not?

From third law of motion, the reaction force exerted by the water on the wall is equal but opposite

Hence, $$F = -\left(-\frac{m}{t} v\right) = \frac{m}{t} v$$

Thus force can be calculated from the product of mass of water striking normally per second and change in velocity. Suppose the water flows out from a pipe at 3 kgs^{-1} and its velocity changes from 5 ms^{-1} to zero on striking the ball, then,

$$\text{Force} = 3\ kgs^{-1} \times (5\ ms^{-1} - 0) = 15 kgms^{-2} = 15\ N$$

Example 3.6: A hose pipe ejects water at a speed of 0.3 ms^{-1} through a hole of area 50 cm^2. If the water strikes a wall normally, calculate the force on the wall, assuming the velocity of the water normal to the wall is zero after striking.

Solution:

$$\left[\begin{array}{c}\text{The volume of water per}\\ \text{second striking the wall}\end{array}\right] = 0.005\ m^2 \times 0.3\ m = 0.0015\ m^3$$

Mass per second striking the wall = volume x density

$$= 0.0015\ m^3 \times 1000\ kgm^{-3} = 1.5\ kg$$

Velocity change of water on striking the wall $= 0.3 ms^{-1} - 0 = 0.3 ms^{-1}$

Force = Momentum change per second

$$= 1.5\ kgs^{-1} \times 0.3\ ms^{-1} = 0.45\ kgms^{-2} = 0.45\ N$$

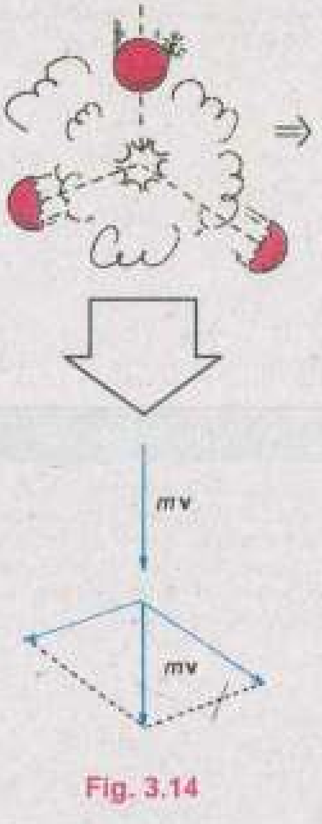


Fig. 3.14

3.10 MOMENTUM AND EXPLOSIVE FORCES

There are many examples where momentum changes are produced by explosive forces within an isolated system. For example, when a shell explodes in mid-air, its fragments fly off in different directions. The total momentum of all its fragments equals the initial momentum of the shell. Suppose a falling bomb explodes into two pieces as shown in Fig. 3.14. The momenta of the bomb fragments combine by vector addition equal to the original momentum of the falling bomb.

Consider another example of bullet of mass *m* fired from a rifle of mass *M* with a velocity **v**. Initially, the total momentum of the bullet and rifle is zero. From the principle of conservation of linear momentum, when the bullet is fired, the total momentum of bullet and rifle still remains zero, since no external force has acted on them. Thus if **v'** is the velocity of the rifle then

$$mv \text{ (bullet)} + Mv' \text{ (rifle)} = 0$$

$$Mv' = -mv \qquad \text{or} \qquad v' = \frac{-mv}{M} \quad \text{..........} \quad (3.20)$$

The momentum of the rifle is thus equal and opposite to that of the bullet. Since mass of rifle is much greater than the bullet, it follows that the rifle moves back or recoils with only a fraction of the velocity of the bullet.

3.11 ROCKET PROPULSION

Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines (Fig. 3.15). The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction. The rocket engines continue to expel gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore work at great heights where very little or no air is present. In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about 10000 kgs^{-1} of fuel and ejects the burnt gases at speeds of over 4000 ms^{-1}. In fact, more than 80% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving others to carry the space craft further up at ever greater speed.

If m is the mass of the gases ejected per second with velocity **v** relative to the rocket, the change in momentum per second of the ejecting gases is mv. This equals the thrust produced by the engine on the body of the rocket. So, the acceleration 'a' of the rocket is

$$\mathbf{a} = \frac{m\mathbf{v}}{M} \quad \text{..........} \quad (3.21)$$

Fig. 3.15

Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals The gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

3.12 PROJECTILE MOTION

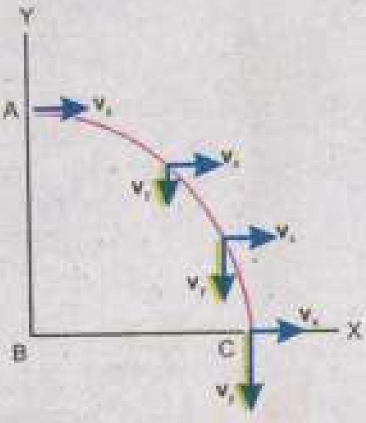


Fig.3.16(a)

Uptill now we have been studying the motion of a particle along a straight line i.e. motion in one dimension. Now we consider the motion of a ball, when it is thrown horizontally from certain height. It is observed that the ball travels forward as well as falls downwards, until it strikes something. Suppose that the ball leaves the hand of the thrower at point A (Fig 3.16 a) and that its velocity at that instant is completely horizontal. Let this velocity be $\mathbf{v}_x$. According to Newton's first law of motion, there will be no acceleration in horizontal direction, unless a horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during flight is the force of gravity. There is no horizontal force acting on it. So its horizontal velocity will remain unchanged and will be $\mathbf{v}_x$, until the ball hits something. The horizontal motion of ball is simple. The ball moves with constant horizontal velocity component. Hence horizontal distance x is given by

$$x = v_x \times t \qquad \text{..........} \qquad (3.22)$$

The vertical motion of the ball is also not complicated. It will accelerate downward under the force of gravity and hence $\mathbf{a} = \mathbf{g}$. This vertical motion is the same as for a freely falling body. Since initial vertical velocity is zero, hence, vertical distance y, using Eq. 3.7, is given by

$$y = \frac{1}{2} gt^2$$

It is not necessary that an object should be thrown with some initial velocity in the horizontal direction. A football kicked off by a player; a ball thrown by a cricketer and a missile fired from a launching pad, all projected at some angles with the horizontal, are called projectiles.

Projectile motion is two dimensional motion under constant acceleration due to gravity.

In such cases, the motion of a projectile can be studied easily by resolving it into horizontal and vertical components which are independent of each other. Suppose that a projectile is fired in a direction angle θ with the horizontal by velocity v_i as shown in Fig. 3.16 (b). Let components of velocity v_i along the horizontal and vertical directions be $v_i \cos\theta$ and $v_i \sin\theta$ respectively. The horizontal acceleration is $a_x = 0$ because we have neglected air resistance and no other force is acting along this direction whereas vertical acceleration $a_y = g$. Hence, the horizontal component v_{ix} remains constant and at any time t, we have

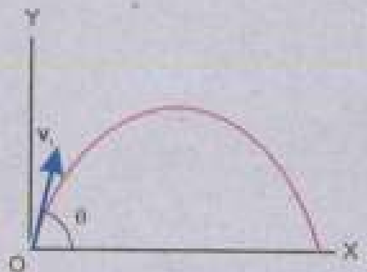


Fig.3.16(b)

$$v_{fx} = v_{ix} = v_i \cos\theta \quad \text{..........} \quad (3.23)$$

Now we consider the vertical motion. The initial vertical component of the velocity is $v_i \sin\theta$ in the upward direction. Using Eq. 3.5 the vertical component v_{fy} of the velocity at any instant t is given by

$$v_{fy} = v_i \sin\theta - gt \quad \text{..........} \quad (3.24)$$

The magnitude of velocity at any instant is

$$v = \sqrt{v_{fx}^2 + v_{fy}^2} \quad \text{..........} \quad (3.25)$$

The angle φ which this resultant velocity makes with the horizontal can be found from

$$\tan\phi = \frac{v_{fy}}{v_{fx}} \quad \text{..........} \quad (3.26)$$

In projectile motion one may wish to determine the height to which the projectile rises, the time of flight and horizontal range. These are described below.

Interesting Information

A photograph of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. At any time the two balls are at the same level, i.e., their vertical displacements are equal.

Height of the Projectile

In order to determine the maximum height the projectile attains, we use the equation of motion

$$2aS = v_f^2 - v_i^2$$

As body moves upward, so $a = -g$, the initial vertical velocity $v_{iy} = v_i \sin\theta$ and $v_{fy} = 0$ because the body comes to rest after reaching the highest point. Since

$$S = \text{height} = h$$

So $$-2gh = 0 - v_i^2 \sin^2\theta$$

or $$h = \frac{v_i^2 \sin^2\theta}{2g} \quad \text{..........} \quad (3.27)$$

Time of Flight

The time taken by the body to cover the distance from the place of its projection to the place where it hits the ground at the same level is called the time of flight.

This can be obtained by taking $S = h = 0$, because the body goes up and comes back to same level, thus covering no vertical distance. If the body is projecting with velocity v making angle θ with a horizontal, then its vertical component will be $v_i \sin\theta$. Hence the equation is

$$S = v_i t + \tfrac{1}{2} g t^2$$

$$0 = v_i \sin\theta \, t - \tfrac{1}{2} g t^2$$

$$t = \frac{2 v_i \sin\theta}{g} \quad \text{..........} \quad (3.28)$$

where t is the time of flight of the projectile when it is projected from the ground as shown in Fig. 3.16 (b).

Point to Ponder

Water is projected from two rubber pipes at the same speed-from one at an angle of 30° and from the other at 60°. Why are the ranges equal?

Range of the Projectile

Maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

To determine the range R of the projectile, we multiply the horizontal component of the velocity of projection with total time taken by the body after leaving the point of projection. Thus

$$R = v_{ix} \times t \qquad \text{using Eq. 3.28}$$

$$R = \frac{v_i \cos\theta \times 2 v_i \sin\theta}{g}$$

$$R = \frac{v_i^2}{g} 2 \sin\theta \cos\theta$$

But, $2 \sin\theta \cos\theta = \sin 2\theta$, thus the range of the projectile depends upon the velocity of projection and the angle of projection.

Therefore, $$R = \frac{v_i^2}{g} \sin 2\theta \text{ .... (3.29)}$$

For the range R to be maximum, the factor $\sin 2\theta$ should have maximum value which is 1 when $2\theta = 90^\circ$ or $\theta = 45^\circ$

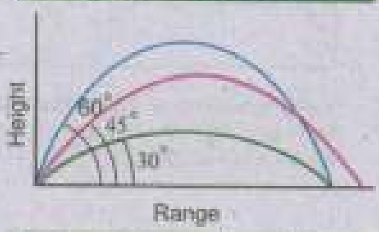
Application to Ballistic Missiles

A ballistic flight is that in which a projectile is given an initial push and is then allowed to move freely due to inertia and under the action of gravity. An un-powered and un-guided missile is called a ballistic missile and the path followed by it is called ballistic trajectory.

As discussed before, a ballistic missile moves in a way that is the result of the superposition of two independent motions: a straight line inertial flight in the direction of the launch and a vertical gravity fall. By law of inertia, an object should sail straight off in the direction thrown, at constant speed equal to its initial speed particularly in empty space. But the downward force of gravity will alter straight path into a curved trajectory. For short ranges and flat Earth approximation, the trajectory is parabolic but the dragless ballistic trajectory for spherical Earth should actually be elliptical. At high speed and for long trajectories the air friction is not negligible and some times the force of air friction is more than gravity. It affects both horizontal as well as vertical motions. Therefore, it is completely unrealistic to neglect the aerodynamic forces.

The shooting of a missile on a selected distant spot is a major element of warfare. It undergoes complicated motions due to air friction and wind etc. Consequently the angle of projection can not be found by the geometry of the situation at the moment of launching. The actual flights of missiles are worked out to high degrees of precision and the result were contained in tabular form. The modified equation of trajectory is too complicated to be discussed here. The ballistic missiles are useful only for short ranges. For long ranges and greater precision, powered and remote control guided missiles are used.

Do You Know ?



For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

Example 3.7: A ball is thrown with a speed of 30 ms^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Solution: Initially

$$v_{ix} = v_i \cos\theta = 30\ ms^{-1} \times \cos 30° = 25.98\ ms^{-1}$$

$$v_{iy} = v_i \sin\theta = 30\ ms^{-1} \times \sin 30° = 15\ ms^{-1}$$

As the time of flight

$$t = \frac{2v_i}{g}\sin\theta$$

So $$t = \frac{2 \times 15\ ms^{-1}}{9.8\ ms^{-2}} = 3.1\ s$$

Height $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

So $$h = \frac{(30\ ms^{-1})^2 \times (0.5)^2}{2 \times 9.8\ ms^{-2}}$$

$$h = 11.5\ m$$

Range $$R = \frac{v_i^2}{g}\sin 2\theta$$

So $$R = \frac{(30\ ms^{-1})^2 \times 0.866}{9.8\ ms^{-2}} = 80m$$

For Your Information

In the presence of air friction the trajectory of a high speed projectile fall short of a parabolic path.

Example 3.8: In example 3.7 calculate the maximum range and the height reached by the ball if the angles of projection are (i) 45° (ii) 60°.

Solution:

(i) Using the equation for height and range we have

height $$h = \frac{v_i^2 \sin^2\theta}{2g}$$

So $$h = \frac{(30\ ms^{-1} \times 0.707)^2}{2 \times 9.8\ ms^{-2}}$$

$$h = 23m$$

Range $R = \frac{v_i^2}{g}\sin 2\theta$

or $R = \frac{v_i^2}{g}\sin 90^\circ$

or $R = \frac{(30\,\text{ms}^{-1})^2}{9.8\,\text{ms}^{-2}} \times 1 = 91.8\text{ m}$

(ii) Using the equation for height and range we have

height $h = \frac{v_i^2 \sin^2\theta}{2g}$

So $h = \frac{(30\,\text{ms}^{-1} \times .866)^2}{2 \times 9.8\,\text{ms}^{-2}}$

or $h = 34.4\text{ m}$

Range $R = \frac{v_i^2}{g}\sin 2\theta$

or $R = \frac{v_i^2}{g}\sin 120^\circ$

or $R = \frac{(30\,\text{ms}^{-1})^2 \times 0.866}{9.8\,\text{ms}^{-2}} = 80\text{m}$

SUMMARY

- Displacement is the change in the position of a body from its initial position to its final position.
- Average velocity is the average rate at which displacement vector changes with time.
- Instantaneous velocity is the velocity at a particular instant of time. When the time interval, over which the average velocity is measured, approaches zero, the average velocity becomes equal to the instantaneous velocity at that instant.

$$\mathbf{v}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{d}}{\Delta t}$$

- Average acceleration is the ratio of the change in velocity $\Delta\mathbf{v}$ that occurs within time interval Δt to that time interval.

- Instantaneous acceleration is the acceleration at a particular instant of time. It is the value obtained from the average acceleration as time interval Δt is made smaller and smaller, approaching zero.

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- The slope of velocity-time graph at any instant represents the instantaneous acceleration at that time.
- The area between velocity-time graph and the time axis is numerically equal to the distance covered by the object.
- Freely falling is a body moving under the influence of gravity alone.
- Acceleration due to gravity near the Earth surface is 9.8 ms^{-2} if air friction is ignored.
- Equations of uniformly accelerated motion are

(i) $v_f = v_i + at$ (ii) $S = \frac{(v_f + v_i)}{2} \cdot t$

(iii) $S = v_i t + \frac{1}{2} at^2$ (iv) $v_f^2 = v_i^2 + 2aS$

- Newton's laws of motion

  1st Law: The velocity of an object will be constant if net force on it is zero.

  2nd Law: An object gains momentum in the direction of applied force, and the rate of change of momentum is proportional to the magnitude of the force.

  3rd Law: When two objects interact, they exert equal and opposite force on each other for the same length of time, and so receive equal and opposite impulses.
- The momentum of an object is the product of its mass and velocity.
- The impulse provided by a force is the product of force and time for which it acts. It equals change in momentum of the object.
- For any isolated system, the total momentum remains constant. The momentum of all bodies in a system add upto the same total momentum at all time.
- Elastic collisions conserve both momentum and kinetic energy. In inelastic collision, some of the energy is transferred by heating and dissipative forces such as friction, air resistance and viscosity, so increasing the internal energy of nearby objects.
- Projectile motion is the motion of particle that is thrown with an initial velocity and then moves under the action of gravity.

QUESTIONS

3.1 What is the difference between uniform and variable velocity? From the explanation of variable velocity, define acceleration. Give SI units of velocity and acceleration.

3.2 An object is thrown vertically upward. Discuss the sign of acceleration due to gravity, relative to velocity, while the object is in air.

3.3 Can the velocity of an object reverse the direction when acceleration is constant? If so, give an example.

3.4 Specify the correct statements:

a. An object can have a constant velocity even its speed is changing.

b. An object can have a constant speed even its velocity is changing.

c. An object can have a zero velocity even its acceleration is not zero.

d. An object subjected to a constant acceleration can reverse its velocity.

3.5 A man standing on the top of a tower throws a ball straight up with initial velocity v_i and at the same time throws a second ball straight downward with the same speed. Which ball will have larger speed when it strikes the ground? Ignore air friction.

3.6 Explain the circumstances in which the velocity **v** and acceleration **a** of a car are

(i) Parallel (ii) Anti-parallel (iii) Perpendicular to one another

(iv) **v** is zero but **a** is not zero (v) **a** is zero but **v** is not zero

3.7 Motion with constant velocity is a special case of motion with constant acceleration. Is this statement true? Discuss.

3.8 Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

3.9 Define impulse and show that how it is related to linear momentum?

3.10 State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

3.11 Explain the difference between elastic and inelastic collisions. Explain how would a bouncing ball behave in each case? Give plausible reasons for the fact that K.E is not conserved in most cases?

3.12 Explain what is meant by projectile motion. Derive expressions for

a. the time of flight b. the range of projectile.

Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

3.13 At what point or points in its path does a projectile have its minimum speed, its maximum speed?

3.14 Each of the following questions is followed by four answers, one of which is correct answer. Identify that answer.

i. What is meant by a ballistic trajectory?

a. The paths followed by an un-powered and unguided projectile.

b. The path followed by the powered and unguided projectile.

c. The path followed by un-powered but guided projectile.

d. The path followed by powered and guided projectile.

ii. What happens when a system of two bodies undergoes an elastic collision?

a. The momentum of the system changes.

b. The momentum of the system does not change.

c. The bodies come to rest after collision.

d. The energy conservation law is violated.

NUMERICAL PROBLEMS

3.1 A helicopter is ascending vertically at the rate of $19.6\ ms^{-1}$. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

(Ans:8.0s)

3.2 Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (ms^{-1})	0	10	20	20	20	20	0
Time (s)	0	30	60	90	120	150	180

Use the graph to calculate

(a) the initial acceleration

(b) the final acceleration and

(c) the total distance travelled by the motorcyclist.

[Ans:(a) $0.33\ ms^{-2}$; (b) $-0.67\ ms^{-2}$; (c) 2.7 km]

3.3 A proton moving with speed of $1.0 \times 10^{7}\ ms^{-1}$ passes through a 0.020 cm thick sheet of paper and emerges with a speed of $2.0 \times 10^{6}\ ms^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

(Ans:- $2.4 \times 10^{17}\ ms^{-2}$, 3.3×10^{-11} s)

3.4 Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the ratio of the magnitude of their velocities after the spring has been released?

(Ans: $\frac{v_1}{v_2} = \frac{m_2}{m_1}$)

3.5 An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through a tiny orifice. The amoeba ejects water with a speed of $1.0 \times 10^{-4} \text{ms}^{-1}$ and at a rate of $1.0 \times 10^{-13} \text{kgs}^{-1}$. Assume that the water is being continuously replenished so that the mass of the amoeba remains the same.

a. If there were no force on amoeba other than the reaction force caused by the emerging jet, what would be the acceleration of the amoeba?

b. If amoeba moves with constant velocity through water, what is force of surrounding water (exclusively of jet) on the amoeba?

[Ans: (a) $1.0 \times 10^{-5} \text{ ms}^{-2}$ (b) 1.0×10^{-17} N]

3.6 A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the firecracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of 3.0 ms^{-1}, how fast will the can be going?

(Ans: 15 ms^{-1})

3.7 An electron ($m = 9.1 \times 10^{-31}$ kg) travelling at $2.0 \times 10^7 \text{ ms}^{-1}$ undergoes a head on collision with a hydrogen atom ($m = 1.67 \times 10^{-27}$ kg) which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of hydrogen atom.

(Ans: $2.2 \times 10^4 \text{ ms}^{-1}$)

3.8 A truck weighing 2500 kg and moving with a velocity of 21 ms^{-1} collides with stationary car weighing 1000 kg. The truck and the car move together after the impact. Calculate their common velocity.

(Ans: 15 ms^{-1})

3.9 Two blocks of masses 2.0 kg and 0.50 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring is 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.

(Ans: 1.4 ms^{-1}, -5.6 ms^{-1})

3.10 A foot ball is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

(Ans: 21ms^{-1})

3.11 A ball is thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How far off it hit the ground and with what velocity?

(Ans: 30m, 25 ms^{-1})

3.12 A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(a) How long was it in air?

(b) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground? (Ans: 10s, 833 m)

3.13 Find the angle of projection of a projectile for which its maximum height and horizontal range are equal. (Ans: 76°)

3.14 Prove that for angles of projection, which exceed or fall short of 45° by equal amounts, the ranges are equal.

3.15 A SLBM (submarine launched ballistic missile) is fired from a distance of 3000km. If the Earth is considered flat and the angle of launch is 45° with horizontal, find the velocity with which the missile is fired and the time taken by SLBM to hit the target.

(Ans: 5.42 kms^{-1}, 13 min)

Chapter 4

WORK AND ENERGY

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand the concept of work in terms of the product of a force and displacement in the direction of the force.
2. Understand and derive the formula Work = **wd** = *mgh* for work done in a gravitational field near Earth's surface.
3. Understand that work can be calculated from area under the force-displacement graph.
4. Relate power to work done.
5. Define power as the product of force and velocity.
6. Quote examples of power from everyday life.
7. Explain the two types of mechanical energy.
8. Understand the work-energy principle.
9. Derive an expression for absolute potential energy.
10. Define escape velocity.
11. Understand that in a resistive medium loss of potential energy of a body is equal to gain in kinetic energy of the body plus work done by the body against friction.
12. Give examples of conservation of energies from everyday life.
13. Describe some non-conventional sources of energy.

Work is often thought in terms of physical or mental effort. In Physics, however, the term work involves two things (i) force (ii) displacement. We shall begin with a simple situation in which work is done by a constant force.

4.1 WORK DONE BY A CONSTANT FORCE

Let us consider an object which is being pulled by a constant force **F** at an angle θ to the direction of motion. The force displaces the object from position A to B through a displacement **d** (Fig. 4.1).

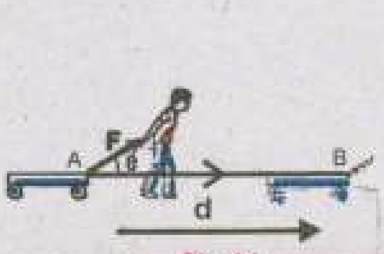


Fig. 4.1

We define work W done by the force $\mathbf{F}$ as the scalar product of $\mathbf{F}$ and $\mathbf{d}$.

$$W = \mathbf{F.d} = Fd\cos\theta \qquad \text{..........} \qquad (4.1)$$

$$= (F\cos\theta)\, d$$

The quantity ($F\cos\theta$) is the component of the force in the direction of the displacement **d**.

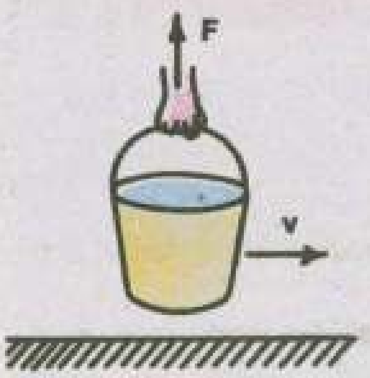


Fig. 4.2(a)

Thus, the work done on a body by a constant force is defined as the product of the magnitudes of the displacement and the component of the force in the direction of the displacement.

Can you tell how much work is being done?

(i) On the pail when a person holding the pail by the force **F** is moving forward (Fig. 4.2 a).

(ii) On the wall (Fig. 4.2 b)?

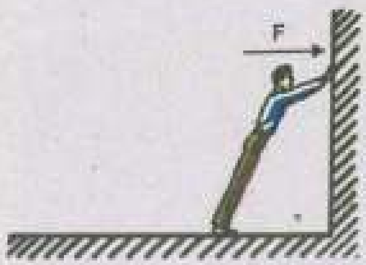


Fig. 4.2(b)

When a constant force acts through a distance d, the event can be plotted on a simple graph (Fig. 4.3). The distance is normally plotted along x-axis and the force along y-axis. In this case as the force does not vary, the graph will be a horizontal straight line. If the constant force **F** (newton) and the displacement **d** (metre) are in the same direction then the work done is Fd (joule). Clearly shaded area in Fig. 4.3 is also Fd. Hence the area under a force- displacement curve can be taken to represent the work done by the force. In case the force **F** is not in the direction of displacement, the graph is plotted between $F\cos\theta$ and d.

From the definition of work, we find that:

(i) Work is a scalar quantity.

(ii) If $\theta < 90^\circ$, work is done and it is said to be positive work.

(iii) If $\theta = 90^\circ$, no work is done.

(iv) If $\theta > 90^\circ$, the work done is said to be negative.

(v) SI unit of work is N m known as joule (J).

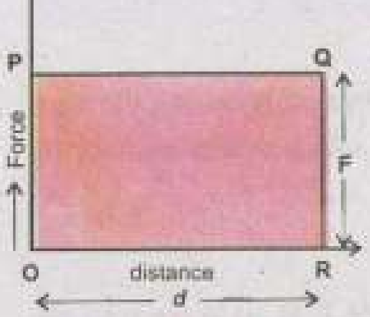


Fig. 4.3

4.2 WORK DONE BY A VARIABLE FORCE

In many cases the force does not remain constant during the process of doing work. For example, as a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre. Similarly, the force exerted by a spring increases with the amount of stretch. How can we calculate the work done in such a situation?

Fig. 4.4 shows the path of a particle in the x-y plane as it moves from point a to point b. The path has been divided into n short intervals of displacements $\Delta \mathbf{d}_1$, $\Delta \mathbf{d}_2$, ..., $\Delta \mathbf{d}_n$ and $\mathbf{F}_1$, $\mathbf{F}_2$, ..., $\mathbf{F}_n$ are the forces acting during these intervals.

Fig. 4.4
A particle acted upon by a variable force, moves along the path shown from point a to point b.

During each small interval, the force is supposed to be approximately constant. So the work done for the first interval can then be written as

$$\Delta W_1 = \mathbf{F}_1 . \Delta \mathbf{d}_1 = F_1 \cos\theta_1 \Delta d_1$$

and in the second interval

$$\Delta W_2 = \mathbf{F}_2 . \Delta \mathbf{d}_2 = F_2 \cos\theta_2 \Delta d_2$$

and so on. The total work done in moving the object can be calculated by adding all these terms.

$$W_{total} = \Delta W_1 + \Delta W_2 + \ldots\ldots\ldots + \Delta W_n$$

$$= F_1 cos\theta_1 \Delta d_1 + F_2 \ cos\theta_2 \Delta d_2 + \ldots\ldots + F_n \ cos\theta_n \ d_n$$

$$W_{total} = \sum_{i=1}^{n} F_i \ cos\theta_i \Delta d_i \qquad \ldots\ldots\ldots \qquad (4.2)$$

We can examine this graphically by plotting $F \cos\theta$ verses d, as shown in Fig. 4.5. The displacement d has been subdivided into n equal intervals. The value of $F \cos\theta$ at the beginning of each interval is indicated in the figure.

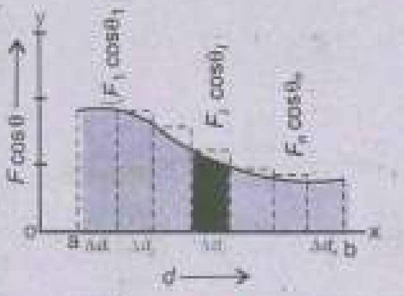


Fig. 4.5

Now the ith shaded rectangle has an area $F_i \ cos\theta_i \Delta d_i$ which is the work done during the ith interval. Thus, the work done given by Eq. 4.2 equals the sum of the areas of all the rectangles. If we subdivide the distance into a large number of intervals so that each Δd becomes very small, the work done given by Eq. 4.2 becomes more accurate. If we let each Δd to approach zero then we obtain an exact result for the work done, such as

$$W_{total} = \underset{\Delta d \to 0}{Limit} \sum_{i=1}^{n} F_i \cos\theta_i \, \Delta d_i \quad \ldots\ldots\ldots \quad (4.3)$$

In this limit Δd approaches zero, the total area of the rectangles (Fig. 4.5) approaches the area between the $F \cos\theta$ curve and d-axis from a to b as shown shaded in Fig. 4.6.

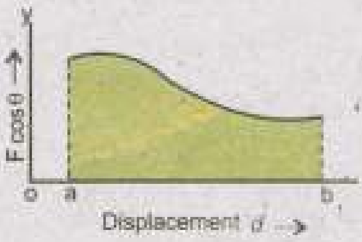


Fig. 4.6

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos\theta$ verses d curve between the two points a and b as shown in Fig. 4.6.

Example 4.1: A force F acting on an object varies with distance x as shown in Fig. 4.7. Calculate the work done by the force as the object moves from $x = 0$ to $x = 6$ m.

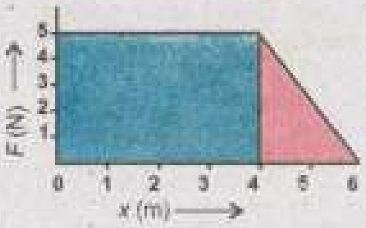


Fig. 4.7

Solution: The work done by the force is equal to the total area under the curve from $x = 0$ to $x = 6$ m. This area is equal to the area of the rectangular section from $x = 0$ to $x = 4$ m, plus the area of triangular section from $x = 4$ m to $x = 6$ m. Hence

Work done represented by the area of rectangle = 4 m x 5 N

= 20 N m = 20 J

Work done represented by the area of triangle = $\frac{1}{2}$ x 2 m x 5 N

= 5 N m = 5 J

Therefore, the total work done = 20 J + 5 J = 25 J

4.3 WORK DONE BY GRAVITATIONAL FIELD

The space around the Earth in which its gravitational force acts on a body is called the gravitational field. When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive. If the displacement is against the gravitational force, the work is negative.

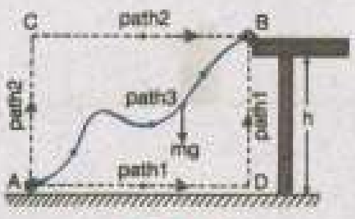


Fig. 4.8

Let us consider an object of mass m being displaced with constant velocity from point A to B along various paths in the presence of a gravitational force (Fig. 4.8). In this case the gravitational force is equal to the weight mg of the object.

The work done by the gravitational force along the path ADB can be split into two parts. The work done along AD is zero, because the weight *mg* is perpendicular to this path, the work done along DB is (−mgh) because the direction of *mg* is opposite to that of the displacement i.e. $\theta = 180^{\circ}$. Hence, the work done in displacing a body from A to B through path 1 is

$$W_{ADB} = 0 + (-mgh) = -mgh$$

If we consider the path ACB, the work done along AC is also (*-mgh*). Since the work done along CB is zero, therefore,

$$W_{ACB} = -mgh + 0 = -mgh$$

Let us now consider path 3, i.e. a curved one. Imagine the curved path, to be broken down into a series of horizontal and vertical steps as shown in Fig. 4.9. There is no work done along the horizontal steps, because *mg* is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements.

Fig. 4.9

A smooth path may be replaced by a series of infinitesimal *x* and *y* displacements. Work is done only during the y displacements.

$$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n)$$

as $(\Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n) = h$

Hence, $W_{AB} = -mgh$

The net amount of work done along AB path is still *(-mgh)*.

We conclude from the above discussion that

Work done in the Earth's gravitational field is independent of the path followed.

Can you prove that the work done along a closed path such as ACBA or ADBA (Fig. 4.8), in a gravitational field is zero?

The field in which the work done be independent of the path followed or work done in a closed path be zero, is called a conservative field.

The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.

4.4 POWER

In the definition of work, it is not clear, whether the same amount of work is done in one second or in one hour. The rate, at which work is done, is often of interest in practical applications.

For Your Information

Conservative Forces
Gravitational force
Elastic spring force
Electric force

Non Conservative forces
Frictional force
Air resistance
Tension in a string
Normal force
Propulsion force of a rocket
Propulsion force of a motor

Power is the measure of the rate at which work is being done.

If work ΔW is done in a time interval Δt, then the average power P_{av} during the interval Δt is defined as

$$P_{av} = \frac{\Delta W}{\Delta t} \quad \text{..........} \quad (4.4)$$

If work is expressed as a function of time, the instantaneous power P at any instant is defined as

$$P = \underset{\Delta t \to 0}{Limit} \frac{\Delta W}{\Delta t} \quad \text{..........} \quad (4.5)$$

Where ΔW is the work done in short interval of time Δt following the instant t.

Power and Velocity

It is, sometimes, convenient to express power in terms of a constant force **F** acting on an object moving at constant velocity **v**. For example, when the propeller of a motor boat causes the water to exert a constant force **F** on the boat, it moves with a constant velocity **v**. The power delivered by the motor at any instant is, then, given by

$$P = \underset{\Delta t \to 0}{Limit} \frac{\Delta W}{\Delta t}$$

we know $\quad \Delta W = \mathbf{F}.\Delta \mathbf{d}$

so $\quad P = \underset{\Delta t \to 0}{Limit} \frac{\mathbf{F}.\Delta \mathbf{d}}{\Delta t}$

Since $\lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t} = v$

Hence, $P = F.v$ .... (4.6)

The SI unit of power is watt, defined as one joule of work done in one second.

Sometimes, for example, in the electrical measurements, the unit of work is expressed as watt second. However, a commercial unit of electrical energy is kilowatt-hour.

One kilowatt hour is the work done in one hour by an agency whose power is one kilowatt.

Therefore, 1 kWh = 1000 W x 3600 s.

or $1\ \text{kWh} = 3.6 \times 10^6 \text{J} = 3.6\ \text{M J}$

Example 4.2: A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Solution: Work done = mgh

$$\text{Power} = \frac{mgh}{t}$$

$$P = \frac{70\ \text{kg} \times 9.8\ \text{ms}^{-2} \times 4.5\ \text{m}}{4\ \text{s}}$$

$$P = 7.7 \times 10^2\ \text{kgm}^2\text{s}^{-3} = 7.7 \times 10^2\ \text{W}$$

For Your Information

Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	1.3x10[illegible]
Car at 90 km h^{-1}	1.1x10[illegible]
Electric heater	2x10[illegible]
Colour Tv	120
Flash light (two cells)	1.5
Pocket calculator	7.5x10[illegible]

4.5 ENERGY

Energy of a body is its capacity to do work. There are two basic forms of energy.

(i) Kinetic energy (ii) Potential energy

The kinetic energy is possessed by a body due to its motion and is given by the formula

$$K.E. = \frac{1}{2}mv^2 \quad .... \quad (4.7)$$

Do You Know?

It takes about 9×10^9 J to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

where m is the mass of the body moving with velocity **v**.

The potential energy is possessed by a body because of its position in a force field e.g. gravitational field or because of its constrained state. The potential energy due to gravitational field near the surface of the Earth at a height h is given by the formula

$$P.E. = mgh \qquad \text{..........} \qquad (4.8)$$

For Your Information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^{9}
Burning 1 litre petrol	5×10^{7}
K.E. of a car at 90 km h^{-1}	1×10^{6}
Running Person at 10 km h^{-1}	3×10^{2}
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

This is called gravitational potential energy. The gravitational P.E. is always determined relative to some arbitrary position which is assigned the value of zero P.E. In the present case, this reference level is the surface of the Earth as position of zero P.E. In some cases a point at infinity from the Earth can also be chosen as zero reference level.

The energy stored in a compressed spring is the potential energy possessed by the spring due to its compressed or stretched state. This form of energy is called the elastic potential energy.

Work-Energy Principle

Whenever work is done on a body, it increases its energy. For example a body of mass m is moving with velocity v_i. A force F acting through a distance d increases the velocity to v_f, then from equation of motion

$$2ad = v_f^2 - v_i^2$$

or

$$d = \frac{1}{2a}(v_f^2 - v_i^2) \qquad \text{..........} \qquad (4.9)$$

From second law of motion

$$F = ma \qquad \text{..........} \qquad (4.10)$$

Tid-bits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Multiplying equations 4.9 and 4.10, we have

$$Fd = \frac{1}{2}m(v_f^2 - v_i^2)$$

or

$$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad \text{..........} \qquad (4.11)$$

where the left hand side of the above equation gives the work done on the body and right hand side gives the increase or change in kinetic energy of the body. Thus

Work done on the body equals the change in its kinetic energy.

This is known as work-energy principle. If a body is raised up from the Earth's surface, the work done changes the gravitational potential energy. Similarly, if a spring is compressed, the work done on it equals the increase in its elastic potential energy.

Absolute Potential Energy

The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero. The relation for the calculation of the work done by the gravitational force or potential energy = *mgh*, is true only near the surface of the Earth where the gravitational force is nearly constant. But if the body is displaced through a large distance in space from, let, point 1 to N (Fig. 4.10) in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance.

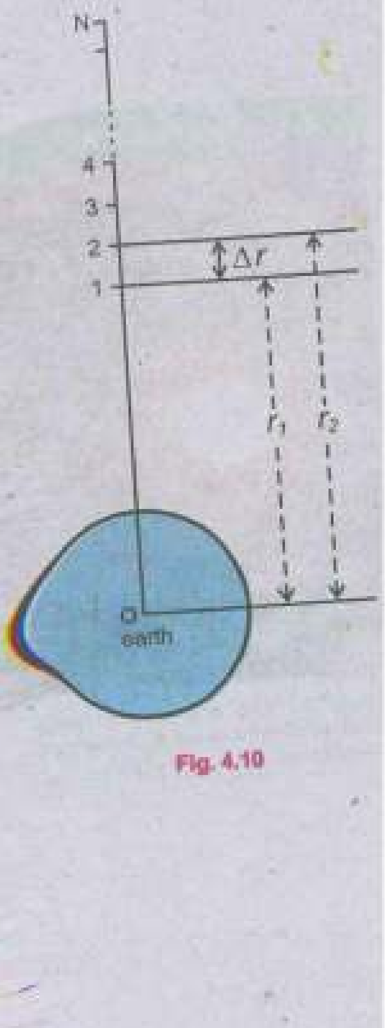


Fig. 4.10

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If r_1 and r_2 are the distances of points 1 and 2 respectively, from the centre O of the Earth (Fig. 4.10.), the work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as below.

The distance between the centre of this step and the centre of the Earth will be

$$r = \frac{r_1 + r_2}{2}$$

if $\quad r_2 - r_1 = \Delta r \quad$ then $\quad r_2 = r_1 + \Delta r$

or $$W_{1\to 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly the work done during the second step in which the body is displaced from point 2 to 3 is

$$W_{2\to 3} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

and the work done in the last step is

$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Hence, the total work done in displacing a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1\to 2} + W_{2\to 3} + \ldots\ldots\ldots + W_{N-1\to N}$$

$$= -G\,Mm\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \ldots\ldots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)\right]$$

On simplification, we get

$$W_{total} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

If the point N is situated at an infinite distance from the Earth, so

$r_N = \infty$, then $\frac{1}{r_N} = \frac{1}{\infty} = 0$

Hence, $$W_{total} = \frac{-GMm}{r_1}$$

Therefore, the general expression for the gravitational potential energy of a body situated at distance r from the centre of Earth is

$$U = \frac{-GMm}{r}$$

This is also known as the absolute value of gravitational potential energy of a body at a distance r from the centre of the Earth.

Tid-bits

More coal has been used since 1945 than was used in the whole of history before that.

Note that when r increases, U becomes less negative i.e., U increases. It means when we raise a body above the surface of the Earth its P.E. increases. The choice of zero point is arbitrary and only the difference of P.E. From one point to another is significant, wether we consider the surface of the Earth or the point at infinity as zero P.E. reference, the change in P.E. as we move a body above the surface of the Earth, will always be positive.

Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth)

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R} \quad \ldots\ldots \quad (4.15)$$

The negative sign shows that the Earth's gravitational field for mass m is attractive. The above expression gives the work or the energy required to take the body out of the Earth's gravitational field, where its potential energy with respect to Earth is zero.

For Your Information

Some Escape speeds (kms^{-1})

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

Escape Velocity

It is our daily life experience that an object projected upward comes back to the ground after rising to a certain height. This is due to the force of gravity acting downward. With increased initial velocity, the object rises to the greater height before coming back. If we go on increasing the initial velocity of the object, a stage comes when it will not return to the ground. It will escape out of the influence of gravity. The initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity.

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of Earth.

$$\text{Initial K.E.} = \frac{1}{2}mv_{esc}^2$$

We know that the work done in lifting a body from Earth's surface to an infinite distance is equal to the increase in its potential energy

$$\text{Incerase in P.E.} = 0-(-G\frac{Mm}{R}) = G\frac{Mm}{R}$$

where M and R are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial K.E. of the body is equal to the increase in P.E. of the body in lifting it up to infinity. Then

$$\frac{1}{2}mv_{esc}^2 = G\frac{Mm}{R}$$

or $$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \ldots\ldots\ldots \quad (4.16)$$

As $$g = \frac{GM}{R^2}$$

Hence, $$v_{esc} = \sqrt{2gR} \quad \ldots\ldots\ldots \quad (4.17)$$

The value of v_{esc} comes out to be approximately 11 kms^{-1}

4.6 INTERCONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

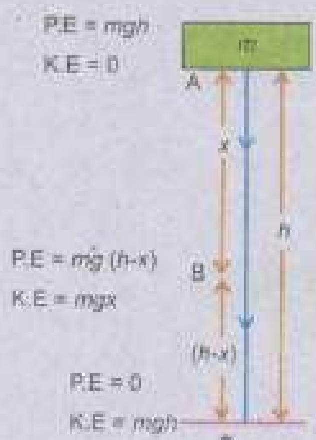


Fig: 4.11

Consider a body of mass m at rest, at a height h above the surface of the Earth as shown in Fig. 4.11. At position A, the body has P.E. $= mgh$ and K.E. $= 0$. We release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

Let us calculate P.E. and K.E. at position B when the body has fallen through a distance x, ignoring air friction.

$$\text{P.E.} = mg\,(h-x)$$

and $$\text{K.E.} = \frac{1}{2}mv_B^2$$

Velocity v_B, at B, can be calculated from the relation,

$$v_f^2 = v_i^2 + 2gS$$

$$v_f = v_B \quad , \quad v_i = 0 \quad , \quad S = x$$

$$v_B^2 = 0 + 2gx = 2gx$$

$$K.E. = \frac{1}{2}m\,(2gx) = mgx$$

Total energy at B = P.E. + K.E.

$$= mg\,(h - x) + mgx = mgh \quad \ldots\ldots \quad (4.18)$$

At position C, just before the body strikes the Earth, P.E. = 0 and K.E. = $\frac{1}{2}mv_C^2$, where v_C can be found out by the following expression.

$$v_C^2 = v_i^2 + 2gh = 2gh \quad as \quad v_i = 0$$

i.e.,
$$K.E. = \frac{1}{2}mv_C^2 = \frac{1}{2}m \times 2gh = mgh$$

Thus at point C, kinetic energy is equal to the original value of the potential energy of the body. Actually when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus we see (Fig. 4.12) that,

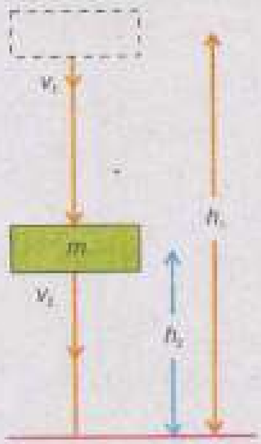


Fig. 4.12

Loss in P.E. = Gain in K.E.

$$mg\,(h_1 - h_2) = \frac{1}{2}m\,(v_2^2 - v_1^2) \quad \ldots\ldots\ldots \quad (4.19)$$

Where v_1 and v_2 are velocities of the body at heights h_1 and h_2 respectively. This result is true only when frictional force is not considered.

If we assume that a frictional force f is present during the downward motion, then a part of P.E. is used in doing work against friction equal to fh. The remaining P.E. = $mgh - fh$ is converted into K.E.

Hence,
$$mgh - fh = \frac{1}{2}mv^2$$

or
$$mgh = \frac{1}{2}mv^2 + fh \quad \ldots\ldots\ldots \quad (4.20)$$

Thus,

Loss in P.E. = Gain in K.E. + Work done against friction.

4.7 CONSERVATION OF ENERGY

The kinetic and potential energies are both different forms of the same basic quantity, i.e. mechanical energy. Total mechanical energy of a body is the sum of the kinetic energy and potential energy. In our previous discussion of a falling body, potential energy may change into kinetic energy and vice versa, but the total energy remains constant. Mathematically,

$$\text{Total Energy} = \text{P.E.} + \text{K.E.} = \text{Constant}$$

This is a special case of the law of conservation of energy which states that:

Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant.

This is one of the basic laws of physics. We daily observe many energy transformations from one form to another. Some forms, such as electrical and chemical energy, are more easily transferred than others, such as heat. Ultimately all energy transfers result in heating of the environment and energy is wasted. For example, the P.E. of the falling object changes to K.E., but on striking the ground, the K.E. changes into heat and sound. If it seems in an energy transfer that some energy has disappeared, the lost energy is often converted into heat. This appears to be the fate of all available energies and is one reason why new sources of useful energy have to be developed.

For Your Information

Source of energy	Original source
Solar	Sun
Bio mass	Sun
Fossil fuels	Sun
Wind	Sun
Waves	Sun
Hydro electric	Sun
Tides	Moon
Geothermal	Earth

Energy Sources

Renewable	Nonrenewable
Hydroelectric	Coal
Wind	Natural gas
Tides	Oil
Geothermal *	Uranium
Biomass	Oil shale
Sunlight	Tar sands
Ethanol/Methanol**	

* Individual fields may run off

**Renewable when made from bio mass

Example 4.3: A brick of mass 2.0 kg is dropped from a rest position 5.0 m above the ground. What is its velocity at a height of 3.0 m above the ground?

Solution: Using Eq. 4.19

$$mg\,(h_1 - h_2) = \frac{1}{2}m\,(v_2^2 - v_1^2)$$

As $v_1 = 0$ and $v_2 = v$

Hence $v = \sqrt{2g(h_1 - h_2)}$

or $v = \sqrt{2 \times 9.8\,\text{ms}^{-2} \times 2.0\text{m}} = 6.3\ \text{m s}^{-1}$

4.8 NON CONVENTIONAL ENERGY SOURCES

These are the energy sources which are not very common these days. However, it is expected that these sources will contribute substantially to the energy demand of the future. Some of these are introduced briefly here.

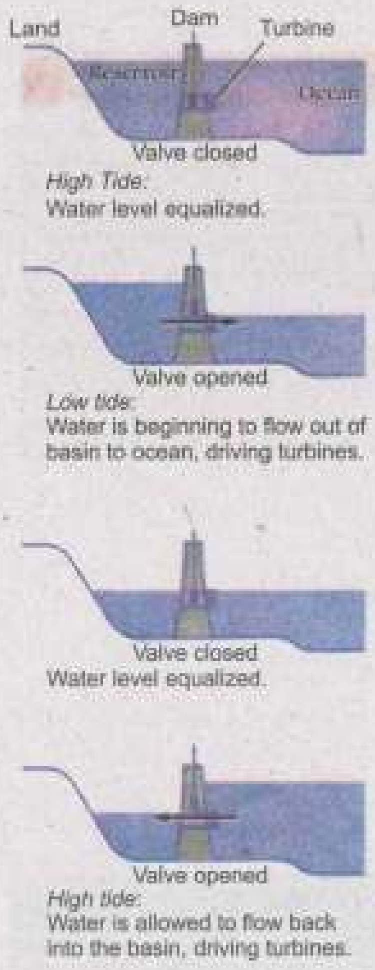


Fig. 4.13

Tidal power plant. Turbines are located inside the dam.

Energy from Tides

One very novel example of obtaining energy from gravitational field is the energy obtained from tides. Gravitational force of the moon gives rise to tides in the sea. The tides raise the water in the sea roughly twice a day. If the water at the high tide is trapped in a basin by constructing a dam, then it is possible to use this as a source of energy. The dam is filled at high tide and water is released in a controlled way at low tide to drive the turbines. At the next high tide the dam is filled again and the in rushing water also drives turbines and generates electricity as shown systematically in the Fig. 4.13.

Energy from Waves

The tidal movement and the winds blowing across the surface of the ocean produce strong water waves. Their energy can be utilized to generate electricity. A method of harnessing wave energy is to use large floats which move up and down with the waves. One such device invented by Professor Salter is known Salter's duck (Fig. 4.14). It consists of two parts (i) Duck float. (ii) Balance float.

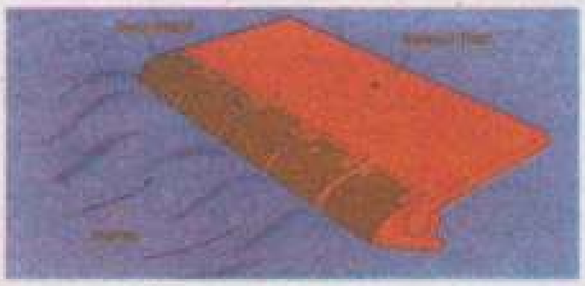


Fig. 4.14

The wave energy makes duck float move relative to the balance float. The relative motion of the duck float is then used to run electricity generators.

Do You Know?

The pull of the Moon does not only pull the sea up and down. This tidal effect can also distort the continents pulling land up and down by as much as 25cm.

Solar Energy

The Earth receives huge amount of energy directly from the Sun each day. Solar energy at normal incidence outside the Earth's atmosphere is about 1.4 kWm^{-2} which is referred as solar constant. While passing through the atmosphere, the total energy is reduced due to reflection, scattering and absorption by dust particles, water vapours and other gases. On a clear day at noon, the intensity of solar energy reaching the Earth's surface is about 1kWm^{-2}. This energy can be used directly to heat water with the help of large solar reflectors and thermal absorbers. It can also be converted to electricity. In one method the flat plate collectors are used for heating water. A typical collector is shown in Fig. 4.15 (a). It has a blackened surface which absorbs energy directly from solar radiation. Cold water passes over the surface and is heated upto about 70°C.

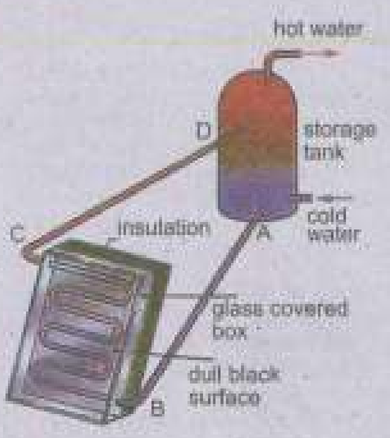


Fig. 4.15(a)

Much higher temperature can be achieved by concentrating solar radiation on to a small surface area by using huge reflectors (mirrors) or lenses to produced steam for running a turbine.

The other method is the direct conversion of sunlight into electricity through the use of semi conductor devices called solar cells also known as photo voltaic cells. Solar cells are thin wafers made from silicon. Electrons in the silicon gain energy from sunlight to create a voltage. The voltage produced by a single voltaic cell is very low. In order to get sufficient high voltage for practical use, a large number of such cells are connected in series forming a solar cell panel.

For cloudy days or nights, electric energy can be stored during the Sun light in Nickel cadmium batteries by connecting them to solar panels. These batteries can then provide power to electrical appliances at nights or on cloudy days.

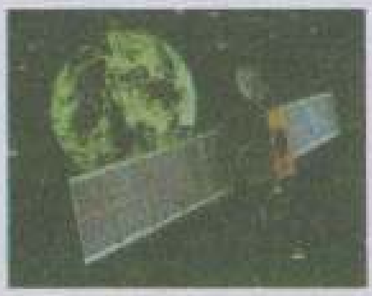


Fig. 4.15(b)

Solar cells, although, are expensive but last a long time and have low running cost. Solar cells are used to power satellites having large solar panels which are kept facing the Sun (Fig. 4.15 b). Other examples of the use of solar cells are remote ground based weather stations and rain forest communication systems. Solar calculators and watches are also in use now-a-days.

Energy From Biomass

For your information

The rapid growth of human population has put a strain on our natural resources. A sustainable society minimizes waste and maximizes the benefit from each resource. Minimizing the use of energy is an other method of conservation. We can save energy by,

(i) turning off lights and electrical appliances when not in use.
(ii) using fluorescent bulbs instead of incandescent bulbs
(iii) using sunlight in offices, commercial centers and houses during daylight hours
(iv) Taking short hot shawers.

Biomass is a potential source of renewable energy. This includes all the organic materials such as crop residue, natural vegetation, trees, animal dung and sewage. Biomass energy or bio conversion refers to the use of this material as fuel or its conversion into fuels.

There are many methods used for the conversion of biomass into fuels. But the most common are

1. Direct combustion 2. Fermentation

Direct combustion method is usually applied to get energy from waste products commonly known as solid waste. It will be discussed in the next section.

Biofuel such as ethanol (alcohol) is a replacement of gasoline. It is obtained by fermentation of biomass using enzymes and by decomposition through bacterial action in the absence of air (oxygen).

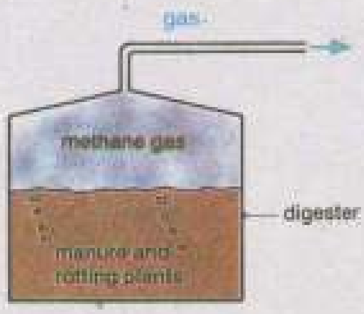


Fig. 4.16

The rotting of biomass in a closed tank called a digester produces Biogas which can be piped out to use for cooking and heating (Fig. 4.16).

The waste material of the process is a good organic fertilizer. Thus, production of biogas provides us energy source and also solves the problem of organic waste disposal.

Energy from Waste Products

Do you know ?

Pollution can be reduced if
(i) People use mass transportation
(ii) Use geothermal, solar, hydroelectrical and wind energy as alternative forms of energy.

Waste products like wood waste, crop residue, and particularly municipal solid waste can be used to get energy by direct combustion. It is probably the most commonly used conversion process in which waste material is burnt in a confined container. Heat produced in this way is directly utilized in the boiler to produce steam that can run turbine generator.

Geothermal Energy

This is the heat energy extracted from inside the Earth in the form of hot water or steam. Heat within the Earth is generated by the following processes.

1. Radioactive Decay

The energy, heating the rocks, is constantly being released by the decay of radioactive elements.

2. Residual Heat of the Earth

At some places hot igneous rocks, usually within 10 km of the Earth's surface, are in a molten and partly molten state. They conduct heat energy from the Earth's interior which is still very hot. The temperature of these rocks is about 200°C or more.

3. Compression of Material

The compression of material deep inside the Earth also causes generation of heat energy.

In some place water beneath the ground is in contact with hot rocks and is raised to high temperature and pressure. It comes to the surface as hot springs, geysers, or steam vents. The steam can be directed to turn turbines of electric generators.

Fig. 4.17

At places water is not present and hot rocks are not very deep, the water is pumped down through them to get steam (Fig. 4.17). The steam then can be used to drive turbines or for direct heating.

An interesting phenomenon of geothermal energy is a geyser. It is a hot spring that discharges steam and hot water, intermittently releasing an explosive column into the air (Fig. 4.18). Most geysers erupt at irregular intervals. They usually occur in volcanic regions. Extraction of geothermal heat energy often occurs closer to geyser sights. This extraction seriously disturbs geyser system by reducing heat flow and aquifer pressure. Aquifer is a layer of rock holding water that allows water to percolate through it with pressure.

Fig. 4.18

SUMMARY

- The work done on a body by a constant force is defined as the product of the magnitude of the displacement and the component of the force in the direction of the displacement.

$$W = \mathbf{F}.\mathbf{d} = Fd\cos\theta$$

- Work done by a variable force is computed by dividing the path into very small displacement intervals and then taking the sum of works done for all such intervals.

$$W = \sum_{i=1}^{n} F_i \cos\theta_i \ \Delta d_i$$

- Graphically, the work done by a variable force in moving a particle between two points is equal to the area under the $F \cos\theta$ verses d curve between these two points.
- When an object is moved in the gravitational field of the Earth, the work is done by the gravitational force. The work done in the Earth's gravitational field is independent of the path followed, and the work done along a closed path is zero. Such a force field is called a conservative field.
- Power is defined as the rate of doing work and is expressed as

$$P = \frac{\Delta W}{\Delta t} \qquad \text{or} \qquad P = \mathbf{F}.\mathbf{v}$$

- Energy of a body is its capacity to do work. The kinetic energy is the energy possessed by a body due to its motion.
- The potential energy is possessed by a body because of its position in a force field.
- The absolute P.E of a body on the surface of Earth is

$$U_g = \frac{-GMm}{R}$$

- The initial velocity of a body with which it should be projected upward so that it does not come back, is called escape velocity.

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

- Some of the non conventional energy sources are

1. Energy from the tides
2. Energy from waves
3. Solar energy
4. Energy from biomass
5. Energy from waste products
6. Geothermal energy

QUESTIONS

4.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

4.2 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m.

4.3 A force F acts through a distance L. The force is then increased to 3 F, and then acts through a further distance of 2 L. Draw the work diagram to scale.

4.4 In which case is more work done? When a 50 kg bag of books is lifted through 50 cm, or when a 50 kg crate is pushed through 2m across the floor with a force of 50 N?

4.5 An object has 1 J of potential energy. Explain what does it mean?

4.6 A ball of mass m is held at a height h_1 above a table. The table top is at a height h_2 above the floor. One student says that the ball has potential energy mgh_1 but another says that it is $mg\,(h_1 + h_2)$. Who is correct?

4.7 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

4.8 What sort of energy is in the following:

a) Compressed spring

b) Water in a high dam

c) A moving car

4.9 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

4.10 A boy uses a catapult to throw a stone which accidentally smashes a green house window. List the possible energy changes.

NUMERICAL PROBLEMS

4.1 A man pushes a lawn mower with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

(Ans: 7.5×10^{2} J)

4.2 A rain drop ($m = 3.35 \times 10^{-5}$ kg) falls vertically at a constant speed under the influence of the forces of gravity and friction. In falling through 100 m, how much work is done by (a) gravity and (b) friction.

[Ans: (a) 0.0328 J (b) – 0.0328 J]

4.3 Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on a table. How much work is required to stack them one on the top of another?

(Ans: 40 J)

4.4 A car of mass 800 kg travelling at 54 kmh^{-1} is brought to rest in 60 metres. Find the average retarding force on the car. What has happened to original kinetic energy?

(Ans: 1500 N)

4.5 A 1000 kg automobile at the top of an incline 10 metre high and 100 m long is released and rolls down the hill. What is its speed at the bottom of the incline if the average retarding force due to friction is 480 N?

(Ans: 10 ms^{-1})

4.6 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than the reservoir, in 20 minutes. If density of water is 1000 kg m^{-3}, find

(a) the increase in P.E.

(b) the power delivered by the pump.

[Ans: (a) 9.8×10^6 J (b) 8.2 kW]

4.7 A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 kmh^{-1}. What power (kW) must the engine develop?

(Ans: 8.9 kW)

4.8 How large a force is required to accelerate an electron ($m = 9.1 \times 10^{-31}$ kg) from rest to a speed of $2.0 \times 10^7 ms^{-1}$ through a distance of 5.0 cm?

(Ans: 3.6×10^{-15} N)

4.9 A diver weighing 750 N dives from a board 10 m above the surface of a pool of water. Use the conservation of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting air friction.

(Ans: 9.9 ms^{-1})

4.10. A child starts from rest at the top of a slide of height 4.0 m. (a) What is his speed at the bottom if the slide is frictionless? (b) if he reaches the bottom, with a speed of 6 ms^{-1}, what percentage of his total energy at the top of the slide is lost as a result of friction?

[Ans: (a) 8.8 ms^{-1} (b) 54%]

Chapter 5

CIRCULAR MOTION

Learning Objectives

At the end of this chapter the students will be able to:

1. Describe angular motion.
2. Define angular displacement, angular velocity and angular acceleration.
3. Define radian and convert an angle from radian measure to degree and vice versa.
4. Use the equation $S = r\theta$ and $v = r\omega$.
5. Describe qualitatively motion in a curved path due to a perpendicular force and understand the centripetal acceleration in case of uniform motion in a circle.
6. Derive the equation $a_c = r\omega^2 = v^2/r$ and $F_c = m\omega^2 r = mv^2/r$
7. Understand and describe moment of inertia of a body.
8. Understand the concept of angular momentum.
9. Describe examples of conservation of angular momentum.
10. Understand and express rotational kinetic energy of a disc and a hoop on an inclined plane.
11. Describe the motion of artificial satellites.
12. Understand that the objects in satellites appear to be weightless.
13. Understand that how and why artificial gravity is produced.
14. Calculate the radius of geo-stationary orbits and orbital velocity of satellites.
15. Describe Newton's and Einstein's views of gravitation.

We have studied velocity, acceleration and the laws of motion, mostly as they are involved in rectilinear motion. However, many objects move in circular paths and their direction is continually changing. Since velocity is a vector quantity, this change of direction means that their velocities are not constant. A stone whirled around by a string, a car turning around a corner and satellites in orbits around the Earth are all examples of this kind of motion.

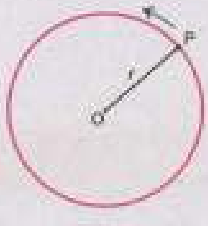


Fig. 5.1(a)

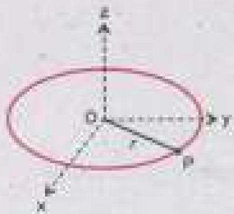


Fig. 5.1(b)

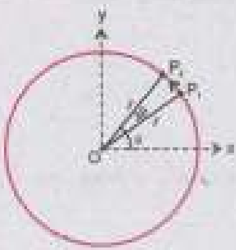


Fig. 5.1(c)

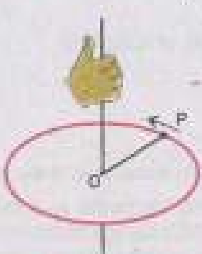


Fig. 5.1(d)

In this chapter we will study, circular motion, rotational motion, moment of inertia, angular momentum and the related topics.

5.1 ANGULAR DISPLACEMENT

Consider the motion of a single particle P of mass m in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of a massless rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. 5.1 (a). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. 5.1 (b). The z-axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis (Fig. 5.1c).

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt.

For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clock wise.

The direction associated with $\Delta\theta$ is along the axis of rotation and is given by right hand rule which states that

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement, as shown in Fig 5.1 (d).

Three units are generally used to express angular displacement, namely degrees, revolution and radian. We are already familiar with the first two. As regards radian which is SI unit, consider an arc of length S of a circle of radius r (Fig 5.2) which subtends an angle θ at the centre of the circle. Its value in radians (rad) is given as

$$\theta = \frac{\text{arclength}}{\text{radius}} \text{ rad}$$

$$\theta = \frac{S}{r} \text{ rad}$$

or $\qquad S = r\theta \qquad$ (where θ is in radian) ..... (5.1)

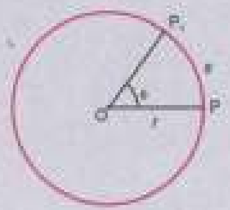


Fig 5.2

If OP is rotating, the point P covers a distance $s = 2\pi r$ in one revolution of P. In radian it would be

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi$$

So $\qquad$ 1 revolution = 2π rad = 360°

Or $\qquad 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$

5.2 ANGULAR VELOCITY

Very often we are interested in knowing how fast or how slow a body is rotating. It is determined by its angular velocity which is defined as the rate at which the angular displacement is changing with time. Referring to Fig. 5.1(c), if $\Delta\theta$ is the angular displacement during the time interval Δt, the average angular velocity ω_{av} during this interval is given by

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad \text{.........} \qquad (5.2)$$

The instantaneous angular velocity ω is the limit of the ratio $\Delta\theta/\Delta t$ as Δt, following instant t, approaches to zero.

Thus $$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} \qquad \text{.........} \qquad (5.3)$$

In the limit when Δt approaches zero, the angular displacement would be infinitesimally small. So it would be a vector quantity and the angular velocity as defined by

Eq.5.3 would also be a vector. Its direction is along the axis of rotation and is given by right hand rule as described earlier.

Angular velocity is measured in radians per second which is its SI unit. Sometimes it is also given in terms of revolution per minute.

5.3 ANGULAR ACCELERATION

When we switch on an electric fan, we notice that its angular velocity goes on increasing. We say that it has an angular acceleration. We define angular acceleration as the rate of change of angular velocity. If ω_i and ω_f are the values of instantaneous velocity of a rotating body at instants t_i and t_f, the average angular acceleration during the interval $t_f - t_i$ is given by

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \quad \text{..........} \quad (5.4)$$

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches zero. Therefore, instantaneous angular acceleration is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \quad \text{..........} \quad (5.5)$$

The angular acceleration is also a vector quantity whose magnitude is given by Eq. 5.5 and whose direction is along the axis of rotation. Angular acceleration is expressed in units of rad s^{-2}.

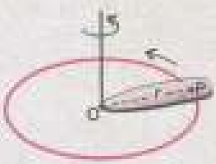


Fig. 5.3

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now we consider the rotation of a rigid body as shown in Fig. 5.3. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation. It is usually referred as reference line. As the body rotates, line OP also rotates with it with the same angular velocity and angular acceleration. Thus the rotation of a rigid body can be described by the rotation of the reference line OP and all the terms that we defined with the help of rotating line OP are also valid for the rotational motion of a rigid body. In future while dealing with rotation of rigid body, we will replace it by its reference line OP.

5.4 RELATION BETWEEN ANGULAR AND LINEAR VELOCITIES

Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. 5.4 (a).

Fig. 5.4(a)

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity v whereas the line OP rotates with angular velocity ω as shown in Fig. 5.4 (b). We are interested in finding out the relation between ω and v. As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, we consider its magnitude only which can also be treated as a scalar.

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP has an angular displacement $\Delta\theta$ radian during this interval. ΔS and $\Delta\theta$ are related by Eq. 5.1.

$$\Delta S = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \qquad \text{..........} \qquad (5.6)$$

In the limit when $\Delta t \rightarrow 0$ the ratio $\Delta S/\Delta t$ represents v, the magnitude of the velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So equation 5.6 becomes

$$v = r\omega \qquad \text{..........} \qquad (5.7)$$

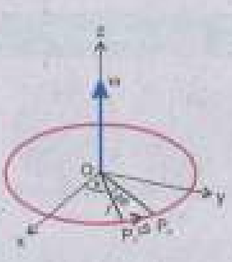


Fig. 5.4(b)

In Fig 5.4 (b), it can be seen that the point P is moving along the arc P_1P_2. In the limit when $\Delta t \rightarrow 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why the linear velocity of the point P is also known as tangential velocity.

Point to Ponder

You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

Similarly Eq 5.7 shows that if the reference line OP is rotating with an angular acceleration α, the point P will also have a linear or tangential acceleration a_t. Using Eq 5.7 it can be shown that the two accelerations are related by

$$a_t = r\alpha \qquad \text{..........} \qquad (5.8)$$

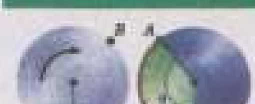
Eqs 5.7 and 5.8 show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body rotating about a fixed axis do have the same angular displacement, angular speed and angular acceleration at any instant. Thus by the use of angular variables we can describe the motion of the entire body in a simple way.

Equations Of Angular Motion

The equations (5.2, 5.3, 5.4 and 5.5) of angular motion are exactly analogous to those in linear motion except that θ, ω and α have replaced S, v and a, respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear	Angular	
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	(5.9)
$2aS = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$	(5.10)
$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$	(5.11)

Do You Know?



As the wheel turns through an angle θ, it lays out a tangential distance $S = r\theta$.

The angular equations 5.9 to 5.11 hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence they can be manipulated as scalars.

Example 5.1: An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Solution: In this problem we have

$\omega_i = 3.0 \text{ rev s}^{-1}$, $\omega_f = 0$, $t = 18.0 \text{ s}$ and $\alpha = ?$, $\theta = ?$

From Eq. 5.4 we have

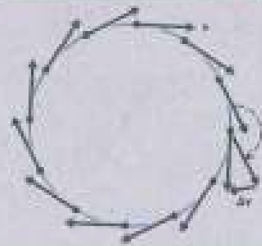
$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{(0 - 3.0) \text{ rev s}^{-1}}{18.0 \text{ s}} = -0.167 \text{ rev s}^{-2}$$

and from Eq 5.11, we have

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 3.0 \text{ rev s}^{-1} \times 18.0 \text{ s} + \frac{1}{2}(-0.167 \text{ rev s}^{-2}) \times (18.0 \text{ s})^2 = 27 \text{ rev}$$

Do You Know?



Direction of motion changes continuously in circular motion.

5.5 CENTRIPETAL FORCE

The motion of a particle which is constrained to move in a circular path is quite interesting. It has direct bearing on the motion of such things as artificial and natural satellites, nuclear particles in accelerators, bodies whirling at the ends of the strings and flywheels spinning on the shafts.

We all know that a ball whirled in a horizontal circle at the end of a string would not continue in a circular path if the string is snapped. Careful observation shows at once that if the string snaps, when the ball is at the point A, in Fig. 5.5 (b), the ball will follow the straight line path AB.

Fig 5.5(a)

The fact is that unless a string or some other mechanism pulls the ball towards the centre of the circle with a force, as shown in Fig. 5.5 (a), ball will not continue along the circular path.

The force needed to bend the normally straight path of the particle into a circular path is called the centripetal force.

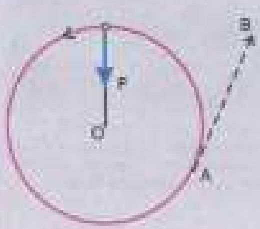


Fig 5.5(b)

If the particle moves from A to B with uniform speed v as shown in Fig. 5.6 (a), the velocity of the particle changes its direction but not its magnitude. The change in velocity is shown in Fig. 5.6 (b). Hence, the acceleration of the particle is

$$a = \frac{\Delta v}{\Delta t}$$

where Δt is the time taken by the particle to travel from A to B. Suppose the velocities at A and B are $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Since the speed of the particle is v, so the time taken to travel a distance s, as shown in Fig. 5.6 (a) is

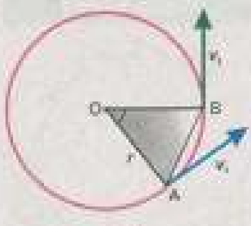


Fig. 5.6(a)

$$\Delta t = \frac{S}{v}$$

so $$a = v\frac{\Delta v}{S} \quad \text{..........} \quad (5.12)$$

Let us now draw a triangle PQR such that PQ is parallel and equal to $\mathbf{v}_1$ and PR is parallel and equal to $\mathbf{v}_2$, as shown in Fig. 5.6 (b). We know that the radius of a circle is perpendicular to its tangent, so OA is perpendicular to $\mathbf{v}_1$ and OB is perpendicular to $\mathbf{v}_2$ (Fig. 5.6 a). Therefore, angle AOB equals the angle QPR between $\mathbf{v}_1$ and $\mathbf{v}_2$. Further, as $v_1 = v_2 = v$ and OA = OB, both triangles are isosceles. From geometry, we know "two isosceles triangles are similar, if the angles between their equal arms are equal". Hence, the triangle OAB of Fig. 5.6 (a) is similar to the triangle PQR of Fig. 5.6 (b). Hence, we can write

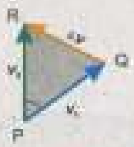


Fig. 5.6(b)

$$\frac{\Delta v}{v} = \frac{AB}{r}$$

If the point B is close to the point A on the circle, as will be the case when $\Delta t \rightarrow 0$, the arc AB is of nearly the same length as the line AB. To that approximation, we can write AB = s, and after substituting and rearranging terms, we have,

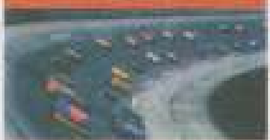
$$\Delta v = S\frac{v}{r}$$

Putting this value for Δv in the Eq. 5.12, we get

$$a = \frac{v^2}{r} \quad \text{..........} \quad (5.13)$$

where a is the instantaneous acceleration. As this acceleration is caused by the centripetal force, it is called the centripetal acceleration denoted by a_c. This acceleration is directed along the radius towards the centre of the circle. In Fig. 5.6 (a) and (b), since PQ is perpendicular to OA and PR is perpendicular to OB, so QR is perpendicular to AB. It may be noted that QR is parallel to the perpendicular bisector of AB. As the acceleration of the object moving in the circle is

Tid-bits

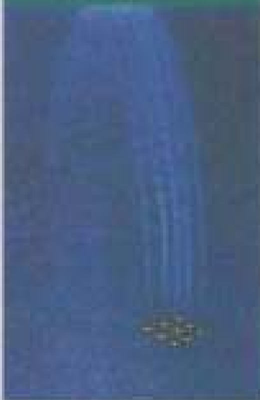


Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

parallel to Δv when AB $\rightarrow 0$, so centripetal acceleration is directed along radius towards the centre of the circle. It can, therefore, be concluded that:

The instantaneous acceleration of an object travelling with uniform speed in a circle is directed towards the centre of the circle and is called centripetal acceleration.

Do You Know?



Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

The centripetal force has the same direction as the centripetal acceleration and its value is given by

$$F_c = ma_c = \frac{mv^2}{r} \qquad \text{.........} \qquad (5.14)$$

In angular measure, this equation becomes

$$F_c = mr\omega^2 \qquad \text{.........} \qquad (5.15)$$

Example 5.2: A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of the circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path?

Solution: The force required is the centripetal force. So

$$F_c = \frac{mv^2}{r} = \frac{1000\,\text{kg} \times 100\,\text{m}^2\text{s}^{-2}}{10\,\text{m}} = 1.0 \times 10^4\,\text{kgms}^{-2} = 1.0 \times 10^4\,\text{N}$$

This force must be supplied by the frictional force of the pavement on the wheels.

Example 5.3: A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 5.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?

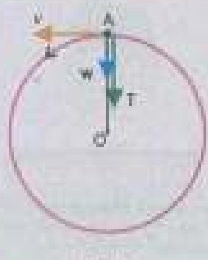


Fig. 5.7

Solution: For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the string and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

$$T + w = \frac{mv^2}{r} \qquad \text{as} \quad w = mg$$

$$\therefore \qquad T = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

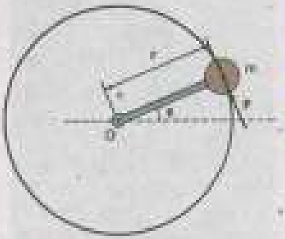


Fig. 5.8

The force F causes a torque about the axis O and gives the mass m an angular acceleration about the pivot point.

5.6 MOMENT OF INERTIA

Consider a mass m attached to the end of a massless rod as shown in Fig. 5.8. Let us assume that the bearing at the pivot point O is frictionless. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence, this will accelerate the mass according to

$$F = ma$$

In doing so the force will cause the mass to rotate about O. Since tangential acceleration a_t is related to angular acceleration α by the equation.

$$a_t = r\alpha$$

so, $$F = mr\alpha$$

As turning effect is produced by torque τ, it would, therefore, be better to write the equation for rotation in terms of torque. This can be done by multiplying both sides of the above equation by r. Thus

$$rF = \tau = \text{torque} = mr^2\alpha$$

which is rotational analogue of the Newton's second law of motion, $F = ma$.

Here F is replaced by τ, a by α and m by mr^2. The quantity mr^2 is known as the moment of inertia and is represented by I. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

Do You Know?

Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means the mass distribution is not uniform. As shown in Fig. 5.9 (a), the rigid body is made up of n small pieces of masses

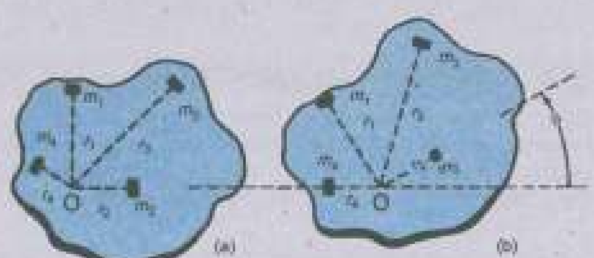


Fig. 5.9
Each small piece of mass within a large, rigid body undergoes the same angular acceleration about the pivot point.

$m_1, m_2, \ldots, m_n$ at distances $r_1, r_2, \ldots, r_n$ from the axis of rotation O. Let the body be rotating with the angular acceleration α so the magnitude of the torque acting on m_1 is

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is

$$\tau_2 = m_2 r_2^2 \alpha_2$$

and so on.

Since the body is rigid, so all the masses are rotating with the same angular asceleration α,

Total torque τ_{total} is then given by

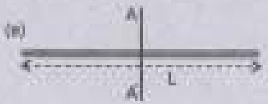
$$\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2)\,\alpha$$

$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

For Your Information

Moments of inertia of various bodies about AA'

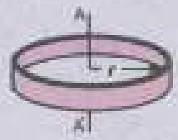
(a)



Thin Rod

$I = \frac{1}{12} mL^2$

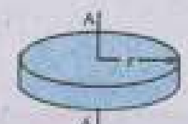
(b)



Thin ring or Hoop

$I = mr^2$

(c)



Solid disc or cylinder

$I = \frac{1}{2} mr^2$

(d)

Sphere

$I = \frac{2}{5} mr^2$

or	$\tau = I\alpha$	....	(5.16)

where I is the moment of inertia of the body and is expressed as

$$I = \sum_{i=1}^{n} m_i r_i^2 \qquad \text{...........} \qquad (5.17)$$

5.7 ANGULAR MOMENTUM

We have already seen that linear momentum plays an important role in translational motion of bodies. Similarly, another quantity known as angular momentum has important role in the study of rotational motion.

Fig. 5.10

A particle is said to posses an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum **L** of a particle of mass m moving with velocity **v** and momentum **p** (Fig. 5.10) relative to the origin O is defined as

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \qquad \text{...........} \qquad (5.18)$$

where **r** is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude is

$$L = rp \sin\theta = m\, rv \sin\theta$$

where θ is the angle between **r** and **p**. The direction of **L** is perpendicular to the plane formed by **r** and **p** and its sense is given by the right hand rule of vector product. SI unit of angular momentum is kg $m^2 s^{-1}$ or J s.

If the particle is moving in a circle of radius r with uniform angular velocity ω, then angle between r and tangential velocity is 90°. Hence

$$L = mrv \sin 90^\circ = mrv$$

But $\qquad v = r\omega$

For Your Information

The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Hence $$L = m r^2 \omega$$

Now consider a symmetric rigid body rotating about a fixed axis through the centre of mass as shown in Fig 5.11. Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω. The magnitude of the angular momentum of the particle of mass m_i is $m_i v_i r_i$ about the origin O. The direction of $\mathbf{L}_i$ is the same as that of ω. Since $v_i = r_i \omega$, the angular momentum of the ith particle is $m_i r_i^2 \omega$. Summing this over all particles gives the total angular momentum of the rigid body.

Fig 5.11

$$L = (\sum_{i=1}^{n} m_i r_i^2)\ \omega = I\omega$$

Where I is the moment of inertia of the rigid body about the axis of rotation.

Physicists usually make a distinction between spin angular momentum (L_s) and orbital angular momentum (L_o). The spin angular momentum is the angular momentum of a spinning body, while orbital angular momentum is associated with the motion of a body along a circular path.

The difference is illustrated in Fig. 5.12. In the usual circumstances concerning orbital angular momentum, the orbital radius is large as compared to the size of the body, hence, the body may be considered to be a point object.

Fig 5.12

Example 5.4: The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7 s).

Solution: To find the Earth's orbital angular momentum we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance of $2\pi r$ in one year, its orbital speed v_o is thus

$$v_o = \frac{2\pi r}{T}$$

Orbital angular momentum of the Earth = $L_o = m v_o r$

$$= \frac{2\pi r^2 m}{T}$$

$$= \frac{2\pi(1.50 \times 10^{11}\ m)^2 \times (6.00 \times 10^{24}\ kg)}{3.16 \times 10^7\ s}$$

$$= 2.67 \times 10^{40}\ kg\ m^2\ s^{-1}$$

The sign is positive because the revolution is counter clockwise.

Fig. 5.13
A man diving from a diving board

5.8 LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots = \text{constant}$$

The law of conservation of angular momentum is one of the fundamental principles of Physics. It has been verified from the cosmological to the submicroscopic level. The effect of the law of conservation of angular momentum is readily apparent if a single isolated spinning body alters its moment of inertia. This is illustrated by the diver in Fig.5.13. The diver pushes off the board with a small angular velocity about a horizontal axis through his centre of gravity. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about this axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed tuck position. As the angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

Hence, the diver must spin faster when moment of inertia becomes smaller to conserve angular momentum. This enables the diver to take extra somersaults.

Point to Ponder

Why does the coasting rotating system slow down as water drips into the beaker?

The angular momentum is a vector quantity with direction along the axis of rotation. In the above example, we discussed the conservation of magnitude of angular momentum. The direction of angular momentum along the axis of rotation also remain fixed. This is illustrated by the fact given below

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

Do You Know?

The law of conservation of angular momentum is important in many sports, particularly in diving, gymnastics and ice-skating.

This fact is of great importance for the Earth as it moves around the Sun. No other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

5.9 ROTATIONAL KINETIC ENERGY

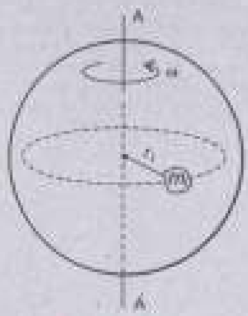


Fig. 5.14

If a body is spinning about an axis with constant angular velocity ω, each point of the body is moving in a circular path and, therefore, has some K.E. To determine the total K.E. of a spinning body, we imagine it to be composed of tiny pieces of mass m_1, m_2, ... If a piece of mass m_i is at a distance r_i from the axis of rotation, as shown in Fig. 5.14, it is moving in a circle with speed

$$v_i = r_i\,\omega$$

Thus the K.E of this piece is

$$K.E_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i(r_i\omega)^2$$

$$= \frac{1}{2}m_i r_i^2\omega^2$$

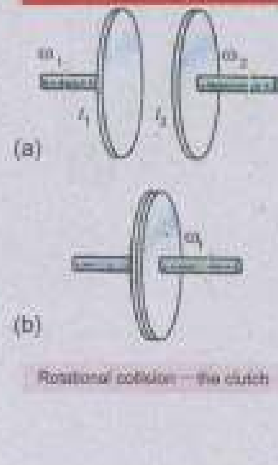
The rotational K.E of the whole body is the sum of the kinetic energies of all the parts. So we have

$$K.E_{rot} = \frac{1}{2}(m_1r_1^2\omega^2 + m_2r_2^2\omega^2 + \dots\dots\dots)$$

$$= \frac{1}{2}(m_1r_1^2 + m_2r_2^2 + \dots\dots\dots)\omega^2$$

We at once recognize that the quantity within the brackets is the moment of inertia I of the body. Hence, rotational kinetic energy is given by

Interesting Information



Rotational collision – the clutch

Where v is the orbital velocity and R is the radius of the Earth (6400 km). From Eq. 5.25 we get,

$$v = \sqrt{gR}$$

$$= \sqrt{9.8\,ms^{-2} \times 6.4 \times 10^{6}\,m}$$

$$= 7.9\,kms^{-1}$$

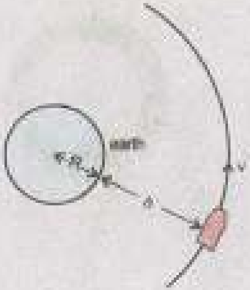


Fig. 5.16

This is the minimum velocity necessary to put a satellite into the orbit and is called critical velocity. The period T is given by

$$T = \frac{2\pi R}{v} = 2 \times 3.14 \times \frac{6400\,km}{7.9\,km\,s^{-1}}$$

$$= 5060s = 84 \text{ min approx.}$$

If, however, a satellite in a circular orbit is at an appreciable distance h above the Earth's surface, we must take into account the experimental fact that the gravitational acceleration decreases inversely as the square of the distance from the centre of the Earth (Fig. 5.16).

The higher the satellite, the slower will the required speed and longer it will take to complete one revolution around the Earth.

Close orbiting satellites orbit the Earth at a height of about 400 km. Twenty four such satellites form the Global Positioning System. An airline pilot, sailor or any other person can now use a pocket size instrument or mobile phone to find his position on the Earth's surface to within 10m accuracy.

Tidbits

The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

5.11 REAL AND APPARENT WEIGHT

We often hear that objects appear to be weightless in a spaceship circling round the Earth. In order to examine the effect in some detail, let us first define, what do we mean by the weight? The real weight of an object is the gravitational pull of the Earth on the object. Similarly the weight of an object on the surface of the Moon is taken to be the gravitational pull of the Moon on the object.

Generally the weight of an object is measured by a spring balance. The force exerted by the object on the scale is equal to the pull due to gravity on the object, i.e., the weight of the object. This is not always true, as will be explained a little later, so we call the reading of the scale as apparent weight.

To illustrate this point, let us consider the apparent weight of an object of mass m, suspended by a string and spring balance, in a lift as shown in Fig. 5.17 (a). When the lift is at rest, Newton's second law tells us that the acceleration of the object is zero, the resultant force on it is also zero. If w is the gravitational force acting on it and T is the tension in the string then we have,

at rest
$a = 0$
$T = w$
Fig. 5.17(a)

$$T - w = ma$$

As $\qquad a = 0$

hence, $\qquad T = w \qquad \text{..........} \qquad (5.26)$

This situation will remain so long as $a = 0$. The scale thus shows the real weight of the object. The weight of the object seems to a person in the lift to vary, depending on its motion.

When the lift is moving upwards with an acceleration a, then

$$T - w = ma$$

or $\qquad T = w + ma \qquad \text{..........} \qquad (5.27)$

the object will then weigh more than its real weight by an amount ma.

acceleration downward
$w - T = ma$
$T = w - ma$
Fig. 5.17(b)

Now suppose, the lift and hence, the object is moving downwards with an acceleration a (Fig. 5.17 b), then we have

$$w - T = ma$$

which shows that

$$T = w - ma \qquad \text{..........} \qquad (5.28)$$

The tension in the string, which is the scale reading, is less than w by an amount ma. To a person in the accelerating lift, the object appears to weigh less than w. Its apparent weight is then ($w - ma$).

Let us now consider that the lift is falling freely under gravity. Then $a = g$, and hence,

$$T = w - mg$$

As the weight w of the body is equal to mg so

$$T = mg - mg = 0$$

The apparent weight of the object will be shown by the scale to be zero.

It is understood from these considerations that apparent weight of the object is not equal to its true weight in an accelerating system. It is equal and opposite to the force required to stop it from falling in that frame of reference.

Do You Know?

Your apparent weight differs from your true weight when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

5.12 WEIGHTLESSNESS IN SATELLITES AND GRAVITY FREE SYSTEM

When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless. It does not matter where the object is, whether it is falling under the force of attraction of the Earth, the Sun, or some distant star.

An Earth's satellite is freely falling object. The statement may be surprising at first, but it is easily seen to be correct. Consider the behaviour of a projectile shot parallel to the horizontal surface of the Earth in the absence of air friction. If the projectile is thrown at successively larger speeds, then during its free fall to the Earth, the curvature of the path decreases with increasing horizontal speeds. If the object is thrown fast enough parallel to the Earth, the curvature of its path will match the curvature of the Earth as shown in Fig. 5.18. In this case the space ship will simply circle round the Earth.

Fig. 5.18

The space ship is accelerating towards the centre of the Earth at all times since it circles round the Earth. Its radial acceleration is simply g, the free fall acceleration. In fact, the space ship is falling towards the centre of the Earth at all the times but due to spherical shape of the Earth, it never strikes the surface of the Earth. Since the space ship is in free fall, all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellite. Such a system is called gravity free system.

5.13 ORBITAL VELOCITY

The Earth and some other planets revolve round the Sun in nearly circular paths. The artificial satellites launched by men also adopt nearly circular course around the Earth. This type of motion is called orbital motion.

Fig. 5.19 shows a satellite going round the Earth in a circular path. The mass of the satellite is m_s and v is its orbital speed. The mass of the Earth is M and r represents the radius of the orbit. A centripetal force $m_s v^2/r$ is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equating the gravitational force to the required centripetal force, gives

$$\frac{Gm_s M}{r^2} = \frac{m_s v^2}{r}$$

or $$v = \sqrt{\frac{GM}{r}} \qquad \text{..........} \qquad (5.29)$$

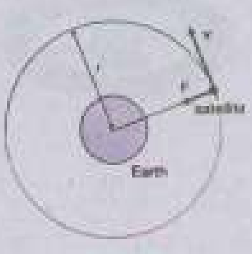


Fig. 5.19

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus any satellite orbiting at distance r from Earth's centre must have the orbital speed given by Eq. 5.29. Any speed less than this will bring the satellite tumbling back to the Earth.

Tid-bits

In 1984, at a height of 100km above Hawaii island with a speed of 29000kmh⁻¹ Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Example 5.6: An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Solution:

As $\quad r = R + h = (6400 + 384000) = 390400$ km

Using $$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\times10^{-11}\,\text{Nm}^2\text{kg}^{-2} \times 6\times10^{24}\,\text{kg}}{390400\text{km}}}$$

$$= 1.01\ \text{kms}^{-1}$$

Also

$$T = \frac{2\pi r}{v} = 2\times3.14\times390400\,\text{km} \times \frac{1}{1.01\text{kms}^{-1}} \times \frac{1\text{day}}{60\times60\times24\text{s}}$$

$$= 27.5\ \text{days}$$

5.14 ARTIFICIAL GRAVITY

Fig. 5.20

In a gravity free space satellite there will be no force that will force any body to any side of the spacecraft. If this satellite is to stay in orbit over an extended period of time, this weightlessness may affect the performance of the astronauts present in that spacecraft. To over come this difficulty, an artificial gravity is created in the spacecraft. This could enable the crew of the space ships to function in an almost normal manner. For this situation to prevail, the space ship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.

Consider a spacecraft of the shape as shown in Fig. 5.20. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed ω, then its angular acceleration a_c is

$$a_c = R\omega^2$$

But $\omega = \frac{2\pi}{T}$ where T is the period of revolution of spaceship

Hence $$a_c = R\frac{(2\pi)^2}{T^2} = R\frac{4\pi^2}{T^2}$$

As frequency $f = 1/T$, therefore $a_c = R\,4\pi^2 f^2$

or $$f^2 = \frac{a_c}{4\pi^2 R} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

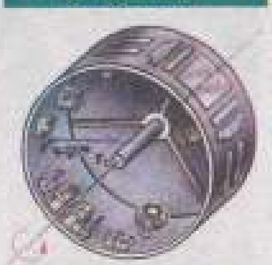
The frequency f is increased to such an extent that a_c equals to g. Therefore,

$$a_c = g$$

and $$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}} \quad \text{..........} \quad (5.30)$$

When the space ship rotates with this frequency, the artificial gravity like Earth is provided to the inhabitants of the space ship.

Do You Know?



The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

5.15 GEOSTATIONARY ORBITS

An interesting and useful example of satellite motion is the geo-synchronous or geo-stationary satellite. This type of satellite is the one whose orbital motion is synchronized with the rotation of the Earth. In this way the synchronous satellite remains always over the same point on the equator as the Earth spins on its axis. Such a satellite is very useful for worldwide communication, weather observations, navigation, and other military uses.

What should the orbital radius of such a satellite be so that it could stay over the same point on the Earth surface? The speed necessary for the circular orbit, given by Eq. 5.29, is

$$v = \sqrt{\frac{GM}{r}}$$

but this speed must be equal to the average speed of the satellite in one day, i.e.,

$$v = \frac{s}{t} = \frac{2\pi r}{T}$$

where T is the period of revolution of the satellite, that is equal to one day. This means that the satellite must move in one complete orbit in a time of exactly one day. As the Earth rotates in one day and the satellite will revolve around the Earth in one day, the satellite at A will always stay over the same point A on the Earth, as shown in Fig. 5.21. Equating the above two equations, we get

Fig. 5.21

$$\frac{2\pi r}{t} = \sqrt{\frac{GM}{r}}$$

Squaring both sides

$$\frac{4\pi^2 r^2}{t^2} = \frac{GM}{r}$$

or

$$r^3 = \frac{GMT^2}{4\pi^2}$$

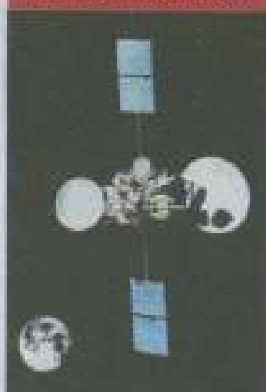
From this we get the orbital radius

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}} \quad \text{..........} \quad (5.31)$$

Substituting the values for the Earth into Eq. 5.31 we get

$$r = 4.23 \times 10^4 \text{ km}$$

For Your Information



A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationary. It is used now for international communications.

which is the orbital radius measured from the centre of the Earth, for a geostationary satellite. A satellite at this height will always stay directly above a particular point on the surface of the Earth. This height above the equator comes to be 36000 km.

5.16 COMMUNICATION SATELLITES

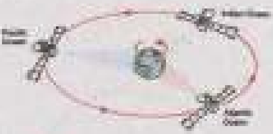


Fig. 5.22

The whole Earth can be covered by just three geo-stationary satellites.

A satellite communication system can be set up by placing several geostationary satellites in orbit over different points on the surface of the Earth. One such satellite covers 120° of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites as shown in Fig. 5.22. Since these geostationary satellites seem to hover over one place on the Earth, continuous communication with any place on the surface of the Earth can be made. Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the Earth. The energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries. You can also pick up the signal from the satellite using a dish antenna on your house. The largest satellite system is managed by 126 countries, International Telecommunication Satellite Organization (INTELSAT). An INTELSAT VI satellite is shown in the Fig.5.23. It operates at microwave frequencies of 4, 6,11 and 14 GHz and has a capacity of 30, 000 two way telephone circuits plus three TV channels.

Fig. 5.23

Communications satellite INTELSAT VI

Example 5.7: Radio and TV signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate. (a) What is the orbital radius for a synchronous satellite? (b) What is its speed?

Do You Know?

1GHz = 10^{9} Hz

Solution:

From Eq. 5.31, $r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$

where $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$, $M = 6.0 \times 10^{24} kg$

and $T = 24 \times 60 \times 60\text{s}$

Therefore, on substitution, we get

a) $$r = \left(\frac{6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2} \times 6.0 \times 10^{24}\,\text{kg} \times (24 \times 60 \times 60\text{s})^2}{4(3.14)^2}\right)^{1/3}$$

$$= 4.23 \times 10^7\,\text{m}$$

b) Substituting the value of r in equation $v = \frac{2\pi r}{T}$

we get,

$$v = \frac{2\pi(4.23 \times 10^7\,\text{m})}{86400\text{s}} = 3.1\,\text{kms}^{-1}$$

5.17 NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION

According to Newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

According to Einstein's theory, space time is curved, especially locally near massive bodies. To visualize this, we might think of space as a thin rubber sheet; if a heavy weight is hung from it, it curves as shown in Fig 5.24. The weight corresponds to a huge mass that causes space itself to curve. Thus, in Einstein's theory we do not speak of the force of gravity acting on bodies; instead we say that bodies and light rays move along geodesics (equivalent to straight lines in plane geometry) in curved space time. Thus, a body at rest or moving slowly near the great mass of Fig. 5.24 would follow a geodesic toward that body.

Einstein's theory gives us a physical picture of how gravity works; Newton discovered the inverse square law of gravity, but explicitly said that he offered no explanation of why gravity should follow an inverse square law. Einstein's theory also says that gravity follows an inverse square law (except in strong gravitational fields), but it tells us why this should be so. That is why Einstein's theory is better than Newton's, even though it includes Newton's theory within itself and

Do You Know?

The gravity can bend light. The gravity of a star could be used to focus light from stars.

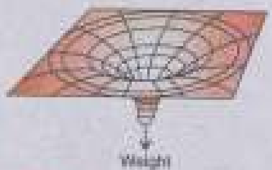


Fig. 5.24

Rubber sheet analogy for curved space-time.

Interesting Information

Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ. Einstein predicted that ϕ = 1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

gives the same answers as Newton's theory everywhere except where the gravitational field is very strong.

Einstein inferred that if gravitational acceleration and inertial acceleration are precisely equivalent, gravity must bend light, by a precise amount that could be calculated. This was not entirely a startling suggestion: Newton's theory, based on the idea of light as a stream of tiny particles, also suggested that a light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exactly twice as great as it is according to Newton's theory. When the bending of starlight caused by the gravity of the Sun was measured during a solar eclipse in 1919, and found to match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

SUMMARY

- Angular displacement is the angle subtended at the centre of a circle by a particle moving along the circumference in a given time.
- SI unit of angular measurement is radian.
- Angular acceleration is the rate of change of angular velocity.
- Relationship between angular and tangential or linear quantities.

  i. $s = r\theta$ ii. $v_T = r\omega$ iii. $a_T = r\alpha$
- The force needed to move a body around a circular path is called centripetal force and is calculated by the expression $F_c = mr\omega^2 = \frac{mv^2}{r}$
- Moment of inertia is the rotational analogue of mass in linear motion. It depends on the mass and the distribution of mass from the axis of rotation.
- Angular momentum is the analogue of linear momentum and is defined as the product of moment of inertia and angular velocity.
- Total angular momentum of all the bodies in a system remains constant in the absence of an external torque.
- Artificial satellites are the objects that orbit around the Earth due to gravity.
- Orbital velocity is the tangential velocity to put a satellite in orbit around the Earth.
- Artificial gravity is the gravity like effect produced in an orbiting spaceship to overcome weightlessness by spinning the spaceship about its own axis.
- Geo-stationary satellite is the one whose orbital motion is synchronized with the rotation of the Earth.
- Albert Einstein viewed gravitation as a space-time curvature around an object.

QUESTIONS

5.1 Explain the difference between tangential velocity and the angular velocity. If one of these is given for a wheel of known radius, how will you find the other?

5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

5.3 What is meant by moment of inertia? Explain its significance.

5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum.

5.5 Show that orbital angular momentum $L_o = mvr$.

5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

5.8 Explain why an object, orbiting the Earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

5.9 When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

5.10 A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

5.11 Why does a diver change his body positions before and after diving in the pool?

5.12 A student holds two dumb-bells with stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumb-bells towards his chest (Fig. 5.25). What will be the effect on rate of rotation?

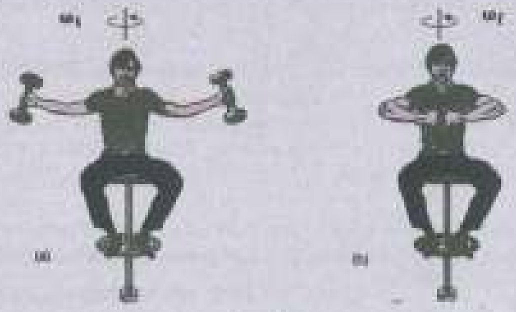


Fig. 5.25

5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission.

NUMERICAL PROBLEMS

5.1 A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8 m.

(Ans: 6.6×10^{-9} rad)

5.2 A gramophone record turntable accelerates from rest to an angular velocity of 45.0 rev min^{-1} in 1.60s. What is its average angular acceleration?

(Ans: 2.95 rad s^{-2})

5.3 A body of moment of inertia I = 0.80 kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

(Ans: 80 Js, 0)

5.4 Consider the rotating cylinder shown in Fig. 5.26. Suppose that m = 5.0 kg, F = 0.60 N and r = 0.20 m. Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder.
(Moment of inertia of cylinder = $\frac{1}{2}mr^2$)

Fig. 5.26

(Ans: 0.12 Nm, 1.2 rad s^{-2})

5.5 Calculate the angular momentum of a star of mass 2.0×10^{30} kg and radius 7.0×10^5 km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

(Ans: 1.4×10^{42} J s, 2.5×10^{36} J)

5.6 A 1000 kg car travelling with a speed of 144 km h^{-1} round a curve of radius 100 m. Find the necessary centripetal force.

(Ans: 1.60×10^4 N)

5.7 What is the least speed at which an aeroplane can execute a vertical loop of 1.0 km radius so that there will be no tendency for the pilot to fall down at the highest point?

(Ans: 99 ms^{-1})

5.8 The Moon orbits the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat the Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^8 m. Radius of the Moon is 1.74×10^6 m.

(Ans: 8.2×10^{-6})

5.9 The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then? (For sphere $I = 2/5\ MR^2$).

(Ans: The Earth would complete its rotation in 6 hours)

5.10 What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} and its radius as 6400 km).

(Ans: 7.4 km s^{-1})

Chapter 6

FLUID DYNAMICS

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand that viscous forces in a fluid cause a retarding force on an object moving through it.
2. Use Stokes' law to derive an expression for terminal velocity of a spherical body falling through a viscous fluid under laminar conditions.
3. Understand the terms steady (laminar, streamline) flow, incompressible flow, non viscous flow as applied to the motion of an ideal fluid.
4. Appreciate that at a sufficiently high velocity, the flow of viscous fluid undergoes a transition from laminar to turbulence conditions.
5. Appreciate the equation of continuity Av = Constant for the flow of an ideal and incompressible fluid.
6. Appreciate that the equation of continuity is a form of the principle of conservation of mass.
7. Understand that the pressure difference can arise from different rates of flow of a fluid (Bernoulli effect).
8. Derive Bernoulli's equation in form $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$.
9. Explain how Bernoulli effect is applied in the filter pump, atomizers, in the flow of air over an aerofoil, Venturimeter and in blood physics.
10. Give qualitative explanations for the swing of a spinning ball.

The study of fluids in motion is relatively complicated, but analysis can be simplified by making a few assumptions. The analysis is further simplified by the use of two important conservation principles: the conservation of mass and the conservation of energy. The law of conservation of mass gives us the equation of continuity while the law of conservation of energy is the basis of Bernoulli's equation. The equation of continuity and the Bernoulli's equation along with their applications in aeroplane and blood circulation are discussed in this chapter.

6.1 VISCOUS DRAG AND STOKES' LAW

The frictional effect between different layers of a flowing fluid is described in terms of viscosity of the fluid. Viscosity measures, how much force is required to slide one layer of the liquid over another layer. Substances that do not flow easily, such as thick tar and honey etc; have large coefficients of viscosity, usually denoted by greek letter 'η'. Substances which flow easily, like water, have small coefficients of viscosity. Since liquids and gases have non zero viscosity, a force is required if an object is to be moved through them. Even the small viscosity of the air causes a large retarding force on a car as it travels at high speed. If you stick out your hand out of the window of a fast moving car, you can easily recognize that considerable force has to be exerted on your hand to move it through the air. These are typical examples of the following fact,

An object moving through a fluid experiences a retarding force called a drag force. The drag force increases as the speed of the object increases.

For Your Information

Viscosities of Liquids and Gases at 30°C

Material	Viscosity 10^{-3} (Nsm^{-2})
Air	0.019
Acetone	0.295
Methanol	0.510
Benzene	0.564
Water	0.801
Ethanol	1.000
Plasma	1.6
Glycerin	6.29

Even in the simplest cases the exact value of the drag force is difficult to calculate. However, the case of a sphere moving through a fluid is of great importance.

The drag force F on a sphere of radius r moving slowly with speed v through a fluid of viscosity η is given by Stokes' law as under.

$$F = 6\pi\eta\, r v \qquad .... \qquad (6.1)$$

At high speeds the force is no longer simply proportional to speed.

6.2 TERMINAL VELOCITY

Consider a water droplet such as that of fog falling vertically, the air drag on the water droplet increases with speed. The droplet accelerates rapidly under the over powering force of gravity which pulls the droplet downward. However, the upward drag force on it increases as the speed of the droplet increases. The net force on the droplet is

$$\text{Net force} = \text{Weight} - \text{Drag force} \quad \ldots\ldots\ldots\ldots \quad (6.2)$$

As the speed of the droplet continues to increase, the drag force eventually approaches the weight in the magnitude. Finally, when the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with constant speed called terminal velocity.

To find the terminal velocity v_t in this case, we use Stokes Law for the drag force. Equating it to the weight of the drop, we have

$$mg = 6\pi\eta r v_t$$

or

$$v_t = \frac{mg}{6\pi\eta r} \quad \ldots\ldots\ldots\ldots \quad (6.3)$$

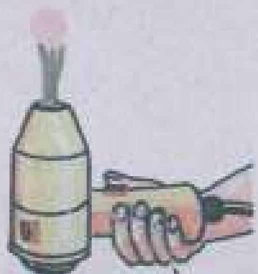
The mass of the droplet is ρV,

where volume $V = \frac{4}{3}(\pi r^3)$

Substituting this value in the above equation, we get

$$v_t = \frac{2gr^2\rho}{9\eta} \quad \ldots\ldots\ldots\ldots \quad (6.4)$$

Can You Do That?



A table tennis ball can be made suspended in the stream of air coming from the nozzle of a hair dryer.

Example 6.1: A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that η for air $= 19 \times 10^{-6}$ kg m^{-1} s^{-1} and density of water $\rho = 1000$ kgm^{-3}.

Solution:

$r = 1.0 \times 10^{-4}$ m, $\rho = 1000$ kgm^{-3}, $\eta = 19 \times 10^{-6}$ kg m^{-1} s^{-1}

Putting the above values in Eq. 6.4

$$v_t = \frac{2 \times 9.8\,\text{ms}^{-2} \times (1 \times 10^{-4}\,\text{m})^2 \times 1000\,\text{kgm}^{-3}}{9 \times 19 \times 10^{-6}\,\text{kgm}^{-1}\text{s}^{-1}}$$

We get Terminal velocity = 1.1 m s^{-1}

6.3 FLUID FLOW

Moving fluids are of great importance. To learn about the behaviour of the fluid in motion, we consider their flow through the pipes. When a fluid is in motion, its flow can be either streamline or turbulent.

(a) Streamlines (laminar flow)

(b) Turbulent flow

Fig. 6.1

The flow is said to be streamline or laminar, if every particle that passes a particular point, moves along exactly the same path, as followed by particles which passed that points earlier.

In this case each particle of the fluid moves along a smooth path called a streamline as shown in Fig. 6.1 (a). The different streamlines can not cross each other. This condition is called steady flow condition. The direction of the streamlines is the same as the direction of the velocity of the fluid at that point. Above a certain velocity of the fluid flow, the motion of the fluid becomes unsteady and irregular.

Under this condition the velocity of the fluid changes abruptly as shown in Fig.6.1 (b). In this case the exact path of the particles of the fluid can not be predicted.

The irregular or unsteady flow of the fluid is called turbulent flow.

For Your Information

Formula One racing cars have a streamlined design.

Dolphins have streamlined bodies to assist their movement in water.

We can understand many features of the fluid in motion by considering the behaviour of a fluid which satisfies the following conditions.

1. The fluid is non-viscous i.e, there is no internal frictional force between adjacent layers of fluid.
2. The fluid is incompressible, i.e., its density is constant.
3. The fluid motion is steady.

6.4 EQUATION OF CONTINUITY

Consider a fluid flowing through a pipe of non-uniform size. The particles in the fluid move along the streamlines in a steady state flow as shown in Fig. 6.2.

In a small time Δt, the fluid at the lower end of the tube moves a distance Δx_1, with a velocity v_1. If A_1 is the area of cross section of this end, then the mass of the fluid contained in the shaded region is:

$$\Delta m_1 = \rho_1 A_1 \Delta x_1 = \rho_1 A_1 v_1 \times \Delta t$$

Where ρ_1 is the density of the fluid. Similarly the fluid that moves with velocity v_2 through the upper end of the pipe (area of cross section A_2) in the same time Δt has a mass

$$\Delta m_2 = \rho_2 A_2 v_2 \times \Delta t$$

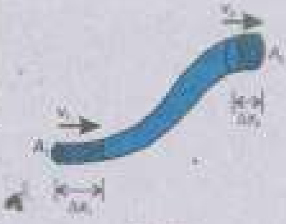


Fig 6.2

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved. That is, the mass that flows into the bottom of the pipe through A_1 in a time Δt must be equal to mass of the liquid that flows out through A_2 in the same time. Therefore,

$$\Delta m_1 = \Delta m_2$$

or $$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This equation is called the equation of continuity. Since density is constant for the steady flow of incompressible fluid, the equation of continuity becomes

$$A_1 v_1 = A_2 v_2 \qquad \text{..........} \qquad (6.5)$$

The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply flow rate.

Tidbits

As the water falls; its speed increases and so its cross sectional area decreases as mandated by the continuity equation.

Example: 6.2: A water hose with an internal diameter of 20 mm at the outlet discharges 30 kg of water in 60 s. Calculate the water speed at the outlet. Assume the density of water is 1000 kgm^{-3} and its flow is steady.

Solution:

Mass flow per second = $\frac{30\text{kg}}{60\text{s}} = 0.5\ \text{kgs}^{-1}$

Cross sectional area $A = \pi r^2$

The mass of water discharging per second through area A is

$$\rho A v = \frac{\text{mass}}{\text{second}}$$

or

$$v = \frac{\text{mass}/\text{second}}{\rho A}$$

$$= \frac{0.5\ \text{kgs}^{-1}}{1000\ \text{kgm}^{-3} \times 3.14 \times (10 \times 10^{-3}\ \text{m})^2}$$

$$= 1.6\ \text{ms}^{-1}$$

6.5 BERNOULLI'S EQUATION

As the fluid moves through a pipe of varying cross section and height, the pressure will change along the pipe. Bernoulli's equation is the fundamental equation in fluid dynamics that relates pressure to fluid speed and height.

In deriving Bernoulli's equation, we assume that the fluid is incompressible, non viscous and flows in a steady state manner. Let us consider the flow of the fluid through the pipe in time t, as shown in Fig. 6.3.

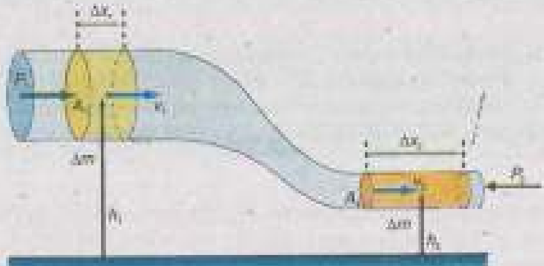


Fig. 6.3

The force on the upper end of the fluid is P_1A_1 where P_1 the pressure and A_1 is the area of cross section at the upper end. The work done on the fluid, by the fluid behind it, in moving it through a distance Δx_1, will be

$$W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$$

Similarly the work done on the fluid at the lower end is

$$W_2 = -F_2\,\Delta x_2 = -P_2 A_2 \Delta x_2$$

Where P_2 is the pressure, A_2 is the area of cross section of lower end and Δx_2 is the distance moved by the fluid in the same time interval t. The work W_2 is taken to be –ive as this work is done against the fluid force.

The net work done = $W = W_1 + W_2$

or $$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \quad \ldots\ldots \quad (6.6)$$

If v_1 and v_2 are the velocities at the upper and lower ends respectively, then

$$W = P_1 A_1 v_1 t - P_2 A_2 v_2 t$$

From equation of continuity (equation 6.5)

$$A_1 v_1 = A_2 v_2$$

Hence, $$A_1 v_1 \times t = A_2 v_2 \times t = V \quad \text{(Volume of fluid under consideration)}$$

So, we have

$$W = (P_1 - P_2)\,V \quad \ldots\ldots \quad (6.7)$$

If m is the mass and ρ is the density then $V = \frac{m}{\rho}$

So equation 6.7 becomes

$$W = (P_1 - P_2)\,\frac{m}{\rho} \quad \ldots\ldots \quad (6.8)$$

Part of this work is utilized by the fluid in changing its K.E. and a part is used in changing its gravitational P.E.

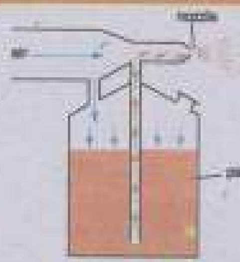
$$\text{Change in K.E.} = \Delta(K.E.) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \quad \ldots \quad (6.9)$$

$$\text{Change in P.E.} = \Delta(P.E.) = mgh_2 - mgh_1 \quad \ldots \quad (6.10)$$

Where h_1 and h_2 are the heights of the upper and lower ends respectively.

Applying, the law of conservation of energy to this volume of the fluid, we get

Interesting Information



A stream of air passing over a tube dipped in a liquid will cause the liquid to rise in the tube as shown. This effect is used in perfume bottles and paint sprayers.

Do You Know?

A chimney works best when it is tall and exposed to air currents, which reduces the pressure at the top and force the upward flow of smoke.

$$(P_1 - P_2)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1 \quad \text{.......} \quad (6.11)$$

rearranging the equation (6.11)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

This is Bernoulli's equation and is often expressed as:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant} \quad \text{..........} \quad (6.12)$$

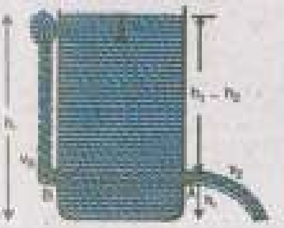


Fig.6.4.

6.6 APPLICATIONS OF BERNOULLI'S EQUATION

Torricelli's Theorem

A simple application of Bernoulli's equation is shown in Fig. 6.4. Suppose a large tank of fluid has two small orifices A and B on it, as shown in the figure. Let us find the speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds v_2 and v_3 will be much larger than the speed v_1 of the top surface of water. We can therefore, take v_1 as approximately zero. Hence, Bernoulli's equation can be written as:

$$P_1 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

But $\qquad P_1 = P_2 =$ atmospheric pressure

Therefore, the above equation becomes

$$v_2 = \sqrt{2g(h_1 - h_2)} \quad \text{..........} \quad (6.13)$$

This is Torricelli's theorem which states that;

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance ($h_1 - h_2$) under the action of gravity.

For your information

A filter pump has a constriction in the centre, so that a jet of water from the tap flows faster here. This cause a drop in pressure near it and air, therefore, flows in from the side tube. The air and water together are expelled through the lower part of the pump.

Notice that the speed of the efflux of liquid is the same as the speed of a ball that falls through a height ($h_1 - h_2$). The top level of the tank has moved down a little and the P.E. has been transferred into K.E. of the efflux of fluid. If the orifice had been pointed upward as at B shown in Fig.6.4, this K.E. would allow the liquid to rise to the level of water tank. In practice, viscous-energy losses would alter the result to some extent.

Relation between Speed and Pressure of the Fluid

A result of the Bernoulli's equation is that the pressure will be low where the speed of the fluid is high. Suppose that water flows through a pipe system as shown in Fig. 6.5. Clearly, the water will flow faster at B than it does at A or C. Assuming the flow speed at A to be 0.20 ms^{-1} and at B to be 2.0 ms^{-1}, we compare the pressure at B with that at A.

Fig. 6.5

Applying Bernoulli's equation and noting that the average P.E. is the same at both places, We have,

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2 \quad \text{.........} \quad (6.14)$$

Substituting $v_A = 0.20\ ms^{-1}$, $v_B = 2.0\ ms^{-1}$

And $\rho = 1000\ kgm^{-3}$

We get $P_A - P_B = 1980\ Nm^{-2}$

This shows that the pressure in the narrow pipe where streamlines are closer together is much smaller than in the wider pipe. Thus,

Where the speed is high, the pressure will be low.

The lift on an aeroplane is due to this effect. The flow of air around an aeroplane wing is illustrated in Fig. 6.6. The wing is designed to deflect the air so that streamlines are closer together above the wing than below it. We have seen in Fig.6.6 that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward.

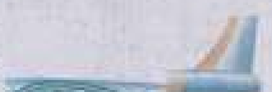


Fig. 6.6

Similarly, when a tennis ball is hit by a racket in such a way that it spins as well as moves forward, the velocity of the air on one side of the ball increases (Fig. 6.7) due to spin and air speed in the same direction as at B and hence, the pressure decreases. This gives an extra curvature to the ball known as swing which deceives an opponent player.

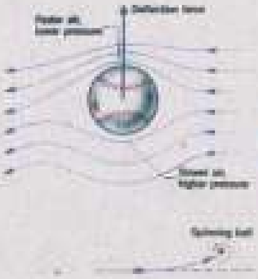


Fig. 6.7

Venturi Relation

If one of the pipes has a much smaller diameter than the other, as shown in Fig. 6.8, we write Bernoulli's equation in a more convenient form. It is assumed that the pipes are horizontal so that ρgh terms become equal and can, therefore, be dropped. Then

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \quad \ldots\ldots \quad (6.15)$$

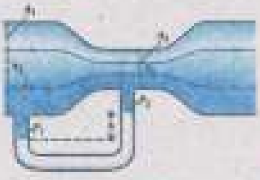


Fig. 6.8

As the cross-sectional area A_2 is small as compared to the area A_1, then from equation of continuity $v_1 = (A_2/A_1)\, v_2$, will be small as compared to v_2. Thus for flow from a large pipe to a small pipe we can neglect v_1 on the right hand side of equation 6.15. Hence,

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 \quad \ldots\ldots\ldots \quad (6.16)$$

This is known as Venturi relation, which is used in Venturi-meter, a device used to measure speed of liquid flow.

Interesting Information

The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through the duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol. The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

Example 6.3: Water flows down hill through a closed vertical funnel. The flow speed at the top is 12.0 cms^{-1}. The flow speed at the bottom is twice the speed at the top. If the funnel is 40.0 cm long and the pressure at the top is $1.013 \times 10^5\ Nm^{-2}$, what is the pressure at the bottom?

Solution: Using Bernoulli's equation

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

Or

$$P_2 = P_1 + \rho gh + \frac{1}{2}\rho(v_1^2 - v_2^2)$$

where $h = h_1 - h_2$ = the length of the funnel

$$P_2 = (1.013 \times 10^5\ Nm^{-2}) + (1000\ kgm^{-3} \times 9.8\ ms^{-2} \times 0.4m)$$
$$+ [\frac{1}{2}(1000kgm^{-3}) \times \{(0.12\ ms^{-1})^2 - (0.24\ ms^{-1})^2\}]$$
$$= 1.05 \times 10^5\ N\ m^{-2}$$

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point the first surges of blood flow through the narrow stricture produces a high flow speed. As a result the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow switches from turbulent to laminar, and the gurgle in the stethoscope disappears. This is the signal to record diastolic pressure.

SUMMARY

- An object moving through a fluid experiences a retarding force known as drag force. It increases as the speed of object increases.
- A sphere of radius r moving with speed v through a fluid of viscosity η experiences a viscous drag force F given by Stokes' law $F = 6\pi\eta r v$.
- The maximum and constant velocity of an object falling vertically downward is called terminal velocity.
- An ideal fluid is incompressible and has no viscosity. Both air and water at low speeds approximate to ideal fluid behaviour.
- In laminar flow, layers of fluid slide smoothly past each other.
- In turbulent flow there is great disorder and a constantly changing flow pattern.
- Conservation of mass in an incompressible fluid is expressed by the equation of continuity $A_1v_1 = A_2v_2$ = constant
- Applying the principles of conservation of mechanical energy to the steady flow of an ideal fluid leads to Bernoulli's equation.

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

- The effect of the decrease in pressure with the increase in speed of the fluid in a horizontal pipe is known as Venturi effect.

Blood Flow

Blood is an incompressible fluid having a density nearly equal to that of water. A high concentration (~50%) of red blood cells increases its viscosity from three to five times that of water. Blood vessels are not rigid. They stretch like a rubber hose. Under normal circumstances the volume of the blood is sufficient to keep the vessels inflated at all times, even in the relaxed state between heart beats. This means there is tension in the walls of the blood vessels and consequently the pressure of blood inside is greater than the external atmospheric pressure. Fig. 6.9 shows the variation in blood pressure as the heart beats. The pressure varies from a high (systolic pressure) of 120 torr (1 torr = 133.3 Nm^{-2}) to a low diastolic pressure) of about 75-80 torr between beats in normal, healthy person. The numbers tend to increase with age, corresponding to the decrease in the flexibility of the vessel walls.

The unit torr or mm of Hg is opted instead of SI unit of pressure because of its extensive use in medical equipments.

An instrument called a sphygmomanometer measures blood pressure dynamically (Fig. 6.10).

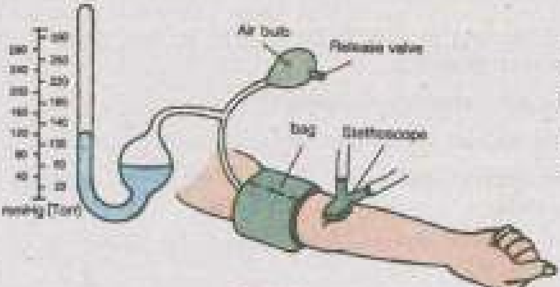


Fig. 6.10

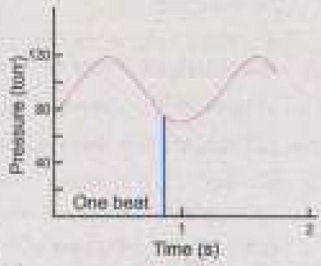


Fig. 6.9

An inflatable bag is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. The effect is to squeeze the arm and compress the blood vessels inside. When the external pressure applied becomes larger than the systolic pressure, the vessels collapse, cutting off the flow of the blood. Opening the release valve on the bag gradually decreases the external pres

QUESTIONS

6.1 Explain what do you understand by the term viscosity?

6.2 What is meant by drag force? What are the factors upon which drag force acting upon a small sphere of radius r, moving down through a liquid, depend?

6.3 Why fog droplets appear to be suspended in air?

6.4 Explain the difference between laminar flow and turbulent flow.

6.5 State Bernoulli's relation for a liquid in motion and describe some of its applications.

6.6 A person is standing near a fast moving train. Is there any danger that he will fall towards it?

6.7 Identify the correct answer. What do you infer from Bernoulli's theorem?

(i) Where the speed of the fluid is high the pressure will be low.

(ii) Where the speed of the fluid is high the pressure is also high.

(iii) This theorem is valid only for turbulent flow of the liquid.

6.8 Two row boats moving parallel in the same direction are pulled towards each other. Explain.

6.9 Explain, how the swing is produced in a fast moving cricket ball.

6.10 Explain the working of a carburetor of a motorcar using by Bernoulli's principle.

6.11 For which position will the maximum blood pressure in the body have the smallest value. (a) Standing up right (b) Sitting (c) Lying horizontally (d) Standing on one's head?

6.12 In an orbiting space station, would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck?

NUMERICAL PROBLEMS

6.1 Certain globular protein particle has a density of 1246 kg m^{-3}. It falls through pure water (η=8.0 x 10^{-4} $Nm^{-2}s$) with a terminal speed of 3.0 cm h^{-1}. Find the radius of the particle.

(Ans: 1.6 x 10^{-6} m)

6.2 Water flows through a hose, whose internal diameter is 1cm at a speed of 1ms^{-1}. What should be the diameter of the nozzle if the water is to emerge at 21ms^{-1}?

(Ans: 0.2 cm)

6.3 The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15m above the point of leak.

a) With what speed does the water rush from the hole?

b) If the hole has an area of 0.060 cm^2, how much water flows out in one second?

(Ans: (a) 17 m s^{-1}, (b) 102 cm^3)

6.4 Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3.0 ms^{-1}, while at another point 3.0 m higher, the speed is 4.0 ms^{-1}. If the pressure is 80 kPa at the lower point, what is pressure at the upper point?

(Ans: 47 kPa)

6.5 An airplane wing is designed so that when the speed of the air across the top of the wing is 450 ms^{-1}, the speed of air below the wing is 410 ms^{-1}. What is the pressure difference between the top and bottom of the wings? (Density of air = 1.29kgm^{-3})

(Ans: 22 kPa)

6.6 The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about 30 cms^{-1}. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8×10^{-4} cm, there are literally millions of them so that their total cross section is about 2000cm^2.

(Ans: $5 \times 10^{-4} ms^{-1}$)

6.7 How large must a heating duct be if air moving 3.0 ms^{-1} along it can replenish the air in a room of 300 m^3 volume every 15 min? Assume the air's density remains constant.

(Ans: Radius = 19 cm)

6.8 An airplane design calls for a "lift" due to the net force of the moving air on the wing of about 1000 Nm^{-2} of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is 160ms^{-1}, what is the required speed over the upper surface to give a "lift" of 1000Nm^{-2}? The density of air is 1.29 kgm^{-3} and assume maximum thickness of wing to be one metre.

(Ans: 165 ms^{-1})

6.9 What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15.0 m?

(Ans: 1.47×10^5 Pa)

Chapter 7

OSCILLATIONS

Learning Objectives

At the end of this chapter the students will be able to:

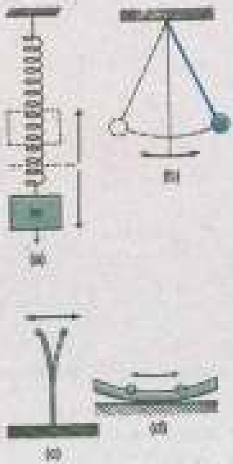
1. Investigate the motion of an oscillator using experimental, analytical and graphical methods.
2. Understand and describe that when an object moves in a circle the motion of its projection on the diameter of the circle is simple harmonic.
3. Show that the motion of mass attached to a spring is simple harmonic.
4. Understand that the motion of simple pendulum is simple harmonic and to calculate its time period.
5. Understand and use the terms amplitude, time period, frequency, angular frequency and phase difference.
6. Know and use of solutions in the form of $x = x_o \cos \omega t$ or $y = y_o \sin \omega t$.
7. Describe the interchange between kinetic and potential energies during SHM.
8. Describe practical examples of free and forced oscillations.
9. Describe practical examples of damped oscillations with particular reference to the effects of the degree of damping and the importance of critical damping in cases such as car suspension system.

Many a times, we come across a type of motion in which a body moves to and fro about a mean position. It is called oscillatory or vibratory motion. The oscillatory motion is called periodic when it repeats itself after equal intervals of time.

Some typical vibrating bodies are shown in Fig. 7.1. It is our common observation that

a) a mass, suspended from a spring, when pulled down and then released, starts oscillating (Fig. 7.1 a).

b) the bob of a simple pendulum when displaced from its rest position and released, vibrates (Fig. 7.1 b).



(Vibrating objects)

Fig 7.1

c) a steel ruler clamped at one end to a bench oscillates when the free end is displaced sideways (Fig. 7.1 c).

d) a steel ball rolling in a curved dish, oscillates about its rest position (Fig. 7.1 d).

Thus to get oscillations, a body is pulled away from its rest or equilibrium position and then released. The body oscillates due to a restoring force. Under the action of this restoring force, the body accelerates and it overshoots the rest position due to inertia. The restoring force then pulls it back. The restoring force is always directed towards the rest position and so the acceleration is also directed towards the rest or mean position.

It is observed that the vibrating bodies produce waves. For example, a violin string produces sound waves in air. There are many phenomena in nature whose explanation requires the understanding of the concepts of vibrations and waves. Although many large structures, such as skyscrapers and bridges, appear to be rigid, they actually vibrate. The architects and the engineers who design and build them, take this fact into account.

7.1 SIMPLE HARMONIC MOTION

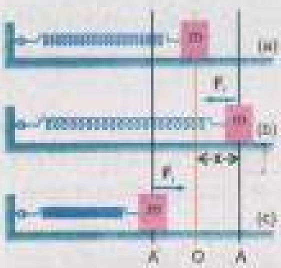


Fig 7.2

Let us consider a mass m attached to one end of an elastic spring which can move freely on a frictionless horizontal surface as shown in Fig. 7.2 (a). When the mass is displaced towards right through a distance **x** (Fig. 7.2 b), the force **F** at that instant is given by Hooke's law $\mathbf{F} = k\mathbf{x}$ where k is a constant known as spring constant. Due to elasticity, spring opposes the applied force which produces the displacement. This opposing force is called restoring force $\mathbf{F}_r$ which is equal and opposite to the applied force within elastic limit of the spring. Hence

$$\mathbf{F}_r = -k\mathbf{x} \quad \text{.........} \quad (7.1)$$

The negative sign indicates that $\mathbf{F}_r$ is directed opposite to **x**, i.e., towards the equilibrium position. Thus we see that in a system obeying Hooke's law, the restoring force $\mathbf{F}_r$ is directly proportional to the displacement **x** of the system from its equilibrium position and is always directed towards it. When the mass is released, it begins to oscillate about the equilibrium position (Fig. 7.2 c). The oscillatory motion taking place under the action of such a restoring force is known as simple harmonic motion (SHM). The acceleration a produced in the mass m due to restoring force can be calculated using second law of motion

$$\mathbf{F} = ma$$

Then, $$-kx = ma$$

or $$a = -\frac{k}{m}x \qquad \text{.........} \qquad (7.2)$$

or $$a \propto -x$$

The acceleration at any instant of a body executing SHM is proportional to displacement and is always directed towards its mean position.

We will now discuss various terms which are very often used in describing SHM.

(i) Instantaneous Displacement and Amplitude of Vibration

It can be seen in Fig. 7.2 that when a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement. It is zero at the instant when the body is at the mean position and is maximum at the extreme positions. The maximum value of displacement is known as amplitude.

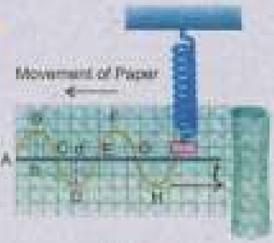


Fig. 7.3

The arrangement shown in Fig. 7.3 can be used to record the variations in displacement with time for a mass-spring system. The strip of paper is moving at a constant speed from right to left, thus providing a time scale on the strip. A pen attached with the vibrating mass records its displacement against time as shown in Fig. 7.3. It can be seen that the curve showing the variation of displacement with time is a sine curve. It is usually known as wave-form of SHM. The points B and D correspond to the extreme positions of the vibrating mass and points A, C and E show its mean position. Thus the line ACE represents the level of mean position of the mass on the strip. The amplitude of vibration is thus a measure of the line Bb or Dd in Fig. 7.3.

(ii) Vibration

A vibration means one complete round trip of the body in motion. In Fig. 7.3, it is the motion of mass from its mean position to the upper extreme position, from upper extreme position to lower extreme position and back to its mean position. In Fig. 7.3, the curve ABCDE correspond to the different positions of the pen during one complete vibration. Alternatively the vibration can also be defined as motion of the body from its one extreme position back to the same extreme position. This will correspond to the portion of curve from points B to F or from points D to H.

(iii) Time Period

It is the time T required to complete one vibration.

(iv) Frequency

Frequency f is the number of vibrations executed by a body in one second and is expressed as vibrations per second or cycles per second or hertz (Hz).

The definitions of T and f show that the two quantities are related by the equation

$$f = \frac{1}{T} \quad \text{..........} \quad (7.3)$$

(v) Angular Frequency

If T is the time period of a body executing SHM, its angular frequency will be

$$\omega = \frac{2\pi}{T} = 2\pi f \quad \text{..........} \quad (7.4)$$

Angular frequency ω is basically a characteristic of circular motion. Here it has been introduced in SHM because it provides an easy method by which the value of instantaneous displacement and instantaneous velocity of a body executing SHM can be computed.

7.2 SHM AND UNIFORM CIRCULAR MOTION

Let a mass m, attached with the end of a vertically suspended spring, vibrate simple harmonically with period T, frequency f and amplitude x_0. The motion of the mass is displayed by the pointer P_1 on the line BC with A as mean position and B, C as extreme positions (Fig. 7.4a). Assuming A as the position of the pointer at $t = 0$, it will move so that it is at B,A,C and back to A at

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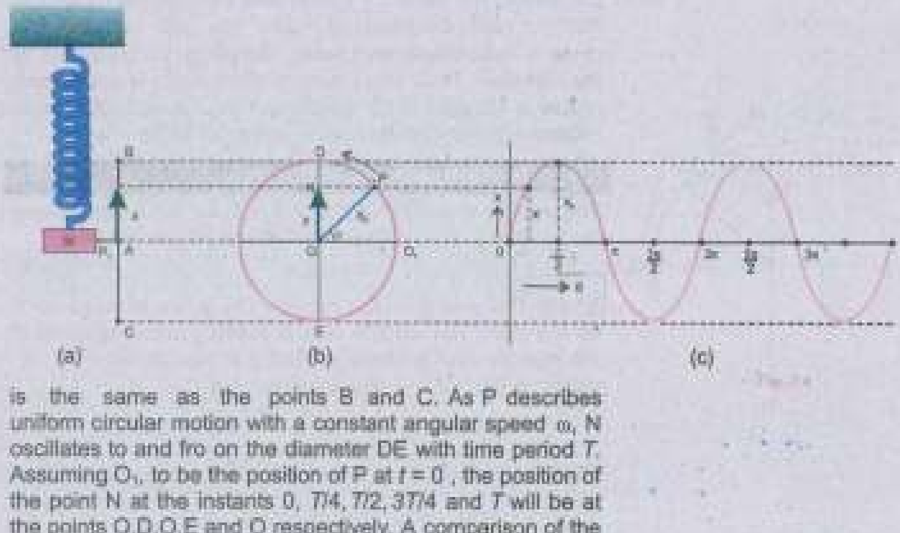
instants $T/4$, $T/2$, $3T/4$ and T respectively. This will complete one cycle of vibration with amplitude of vibration being x_o = AB = AC.

The concept of circular motion is introduced by considering a point P moving on a circle of radius x_o, with a uniform angular frequency $\omega = 2\pi/T$, where T is the time period of the vibration of the pointer. It may be noted that the radius of the circle is equal to the amplitude of the pointer's motion. Consider the motion of the point N, the projection of P on the diameter DE drawn parallel to the line of vibration of the pointer in Fig. 7.4 (b). Note that the level of points D and E



(a) (b) (c)

is the same as the points B and C. As P describes uniform circular motion with a constant angular speed ω, N oscillates to and fro on the diameter DE with time period T. Assuming O_1, to be the position of P at $t = 0$, the position of the point N at the instants 0, $T/4$, $T/2$, $3T/4$ and T will be at the points O,D,O,E and O respectively. A comparison of the motion of N with that of the pointer P_1 shows that it is a replica of the pointer's motion. Thus the expressions of displacement, velocity and acceleration for the motion of N also hold good for the pointer P_1, executing SHM.

(i) Displacement

Referring to Fig. 7.4 (b), if we count the time $t = 0$ from the instant when P is passing through O_1, the angle which the radius OP sweeps out in time t is $\angle O_1OP = \theta = \omega t$. The displacement x of N at the instant t will be

$$x = ON = OP \sin \angle O_1OP$$

or $x = x_0 \sin\theta$

or $x = x_0 \sin \omega t$ .... (7.5)

This will be also the displacement of the pointer P_1 at the instant t.

The value of x as a functions of θ is shown in Fig. 7.4 (c). This is the wave-form of SHM. In Fig. 7.3, the same wave-form was traced experimentally but here, we have traced it theoretically by linking SHM with circular motion through the concept of angular frequency. The angle θ gives the states of the system in its vibrational cycle. For example, at the start of the cycle $\theta = 0$. Half way through the cycle, is 180° (π radians). When $\theta = 270°$ (or $3\pi/2$ radians), the cycle is three-fourth completed. We call θ as the phase of the vibration. Thus when quarter of the cycle is completed, phase of vibration is 90° (or $\pi/2$ radian). Thus phase is also related with the circular motion aspect of SHM.

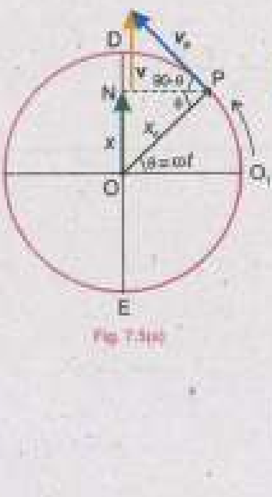


Fig 7.5(a)

(ii) Instantaneous Velocity

The velocity of point P, at the instant t, will be directed along the tangent to the circle at P and its magnitude will be

$$v_P = x_0\omega \quad .... \quad (7.6)$$

As the motion of N on the diameter DE is due to motion of P on the circle, the velocity of N is actually the component of the velocity v_P in a direction parallel to the diameter DE. As shown in Fig. 7.5 (a), this component is

$$v_P \sin(90° - \theta) = v_P \cos\theta = x_0 \omega \cos\theta.$$

Thus the magnitude of the velocity of N or its speed v is

$$v = x_0 \omega \cos\theta = x_0 \omega \cos \omega t \quad .... \quad (7.7)$$

The direction of the velocity of N depends upon the value of the phase angle θ. When θ is between 0° to 90° the direction is from O to D, for θ between 90° to 270°, its direction is from D to E. When θ is between 270° to 360°, the direction of motion is from E to D.

From Fig. 7.5, $\cos\theta = \cos\angle NPO = NP/OP = \dfrac{\sqrt{x_0^2 - x^2}}{x_0}$.

Substituting the value of $\cos\theta$ in Eq. 7.7

$$v = \frac{x_0\omega}{x_0}\sqrt{x_0^2 - x^2} = \omega\sqrt{x_0^2 - x^2} \quad .... \quad (7.8)$$

As the motion of N on the diameter DE is just the replica of the motion of the pointer executing SHM (Fig. 7.4), so velocity of the point P or the velocity of any body executing SHM is given by equations 7.7 and 7.8 in terms of the angular frequency ω. Eq. 7.8 shows that at the mean position, where $x = 0$, the velocity is maximum and at the extreme positions where $x = x_o$, the velocity is zero.

(iii) Acceleration in Terms of ω

When the point P is moving on the circle, it has an acceleration $a_p = x_o\omega^2$, always directed towards the centre O of the circle.

At instant t, its direction will be along PO. The acceleration of the point N will be component of the acceleration a_p along the diameter DE on which N moves due to motion of P. As shown in Fig. 7.5 (b), the value of this component is

$$a_p \sin\theta = x_o\omega^2 \sin\theta$$

Thus the acceleration a of N is $a = x_o \omega^2 \sin\theta$

and it is directed from N to O, i.e., directed towards the mean position O (Fig. 7.5 b). In this figure $\sin\theta = ON/OP = x/x_o$. Therefore,

$$a = x_o\omega^2 \times \frac{x}{x_o} = \omega^2 x$$

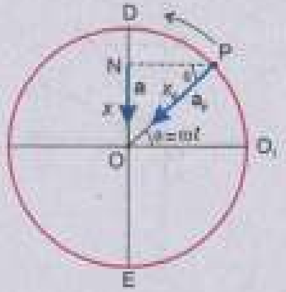


Fig. 7.5(b)

Comparison of Fig. 7.5 (b) and 7.4 (b) shows that the direction of acceleration **a** and displacement **x** are opposite. Considering the direction of **x** as reference, the acceleration **a** will be represented by

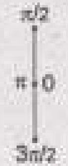
$$\mathbf{a} = -\omega^2 \mathbf{x} \quad \text{.........} \quad (7.9)$$

Eq. 7.9 shows that the acceleration is proportional to the displacement and is directed towards the mean position which is the characteristic of SHM. Thus the point N is executing SHM with the same amplitude, period and instantaneous displacement as the pointer P_1. This confirms our assertion that the motion of N is just a replica of the pointer's motion.

7.3 PHASE

Equations 7.5 and 7.7 indicate that displacement and velocity of the point executing SHM are determined by the angle $\theta = \omega t$. Note that this angle is obtained when SHM is related with circular motion. It is the angle which the rotating radius OP makes with the reference direction OO_1 at any instant t (Fig. 7.4 b).

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of motion of the point executing SHM is known as phase.



The phase determines the state of motion of the vibrating point. If a body starts its motion from mean position, its phase at this point would be 0. Similarly at the extreme positions, its phase would be $\pi/2$.

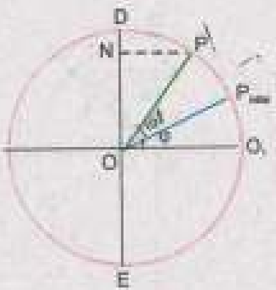


Fig. 7.6 (a)

In Fig. 7.4 (b), we have assumed that to start with at $t = 0$, the position of the rotating radius OP is along OO_1 so that the point N is at its mean position and the displacement at $t = 0$, is zero. Thus it represents a special case. In general at $t = 0$, the rotating radius OP can make any angle φ with the reference OO_1 as shown in Fig. 7.6 (a). In time t, the radius will rotate by ωt. So now the radius OP would make an angle $(\omega t + \varphi)$ with OO_1 at the instant t and the displacement ON $= x$ at instant t would be given by

$$ON = x = OP \sin(\omega t + \varphi)$$

$$= x_0 \sin(\omega t + \varphi) \quad \text{.........} \quad (7.10)$$

Now the phase angle is $\omega t + \varphi$ i.e.,

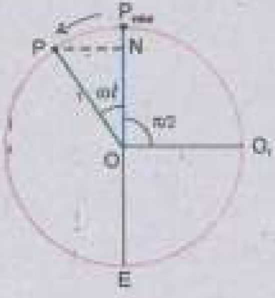
$$\theta = \omega t + \varphi$$

when $t = 0$, $\theta = \varphi$. So φ is the initial phase. If we take initial phase as $\pi/2$ or 90°, the displacement as given by Eq 7.10 is

$$x = x_0 \sin(\omega t + 90^\circ)$$

$$= x_0 \cos \omega t \quad \text{.........} \quad (7.11)$$

Thus Eq. 7.11 also gives the displacement of SHM, but in this case the point N is starting its motion from the extreme position instead of the mean position as shown in Fig. 7.6 (b).



7.4 A HORIZONTAL MASS SPRING SYSTEM

Practically, for a simple harmonic system, consider again the vibrating mass attached to a spring as shown in Fig. 7.2 (a, b and c) whose acceleration at any instant is given by Eq. 7.2 which is

$$a = -\frac{k}{m}x$$

As k and m are constant, we see that the acceleration is proportional to displacement x, and its direction is towards the mean position. Thus the mass m executes SHM between A and A' with x_0 as amplitude. Comparing the above equation with Eq. 7.9, the vibrational angular frequency is

$$\omega = \sqrt{\frac{k}{m}} \quad \text{..........} \quad (7.12)$$

The time period of the mass is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{..........} \quad (7.13)$$

The instantaneous displacement x of the mass as given by Eq. 7.5 is

$$x = x_0 \sin \omega t$$

$$x = x_0 \sin\sqrt{\frac{k}{m}}\,t \quad \text{..........} \quad (7.14)$$

The instantaneous velocity v of the mass m as given by Eq. 7.8 is

$$v = \omega\sqrt{x_0^2 - x^2} = \sqrt{\frac{k}{m}(x_0^2 - x^2)}$$

$$= x_0\sqrt{\frac{k}{m}\left(1 - \frac{x^2}{x_0^2}\right)} \quad \text{..........} \quad (7.15)$$

Eq 7.15 shows that the velocity of the mass gets maximum equal to v_0, when $x = 0$. Thus

$$v_0 = x_0\sqrt{\frac{k}{m}} \quad \text{..........} \quad (7.16)$$

then $$v = v_o\sqrt{1-\frac{x^2}{x_o^2}}$$ .... (7.17)

The formula derived for displacement and velocity are also valid for vertically suspended mass-spring system provided air friction is not considered.

Example 7.1: A block weighing 4.0 kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.50 kg body is hung from the same spring. If the spring is now stretched and then released, what is its period of vibration?

Solution:

Applied stretching force $F = kx$ or $k = \frac{F}{x}$

$$F = mg = 4\text{ kg} \times 9.8\text{ ms}^{-2} = 39.2\text{ kgms}^{-2} = 39.2\text{ N}$$

$x = 0.16\text{m}$, $$k = \frac{4\text{ kg} \times 9.8\text{ ms}^{-2}}{0.16\text{ m}} = 245\text{ kg s}^{-2}$$

Now time period $$T = 2\pi\sqrt{\frac{m}{k}}$$

or $$T = 2\pi\sqrt{\frac{0.5\text{ kg}}{245\text{ kgs}^{-2}}} = 0.28\text{ s}$$

Fig 7.7

7.5 SIMPLE PENDULUM

A simple pendulum consists of a small heavy mass m suspended by a light string of length l fixed at its upper end, as shown in Fig. 7.7. When such a pendulum is displaced from its mean position through a small angle θ to the position B and released, it starts oscillating to and fro over the same path. The weight mg of the mass can be resolved into two components; $mg \sin\theta$ along the tangent at B and $mg \cos\theta$ along CB to balance the tension of the string. The restoring force at B will be

$$F = -mg\sin\theta$$

When θ is small, $\sin\theta \approx \theta$

So $\qquad F = ma = -mg\theta \qquad \ldots\ldots \qquad (7.18)$

Or $\qquad a = -g\theta$

But $\qquad \theta = \dfrac{\text{Arc AB}}{l}$

When θ is small $\quad$ Arc AB = OB = x, hence $\theta = \dfrac{x}{l}$

Thus, $\qquad a = -\dfrac{gx}{l} \qquad \ldots\ldots \qquad (7.19)$

At a particular place 'g' is constant and for a given pendulum 'l' is also a constant.

Therefore, $\qquad \dfrac{g}{l} = k \qquad$ (a constant)

and the motion of the simple pendulum is simple harmonic. Comparing Eq. 7.19 with Eq. 7.9

$$\omega = \sqrt{\frac{g}{l}}$$

As time period $\qquad T = \dfrac{2\pi}{\omega}$

Hence $\qquad T = 2\pi\sqrt{\dfrac{l}{g}} \qquad \ldots\ldots \qquad (7.20)$

This shows that the time period depends only on the length of the pendulum and the acceleration due to gravity. It is independent of mass.

Example 7.2: What should be the length of a simple pendulum whose period is 1.0 second at a place where $g = 9.8\ ms^{-2}$? What is the frequency of such a pendulum?

Solution:

Time period, $\qquad T = 2\pi\sqrt{\dfrac{l}{g}}$

$T = 1.0\ s \qquad , \qquad g = 9.8\ ms^{-2}$

Squaring both sides

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$l = \frac{gT^2}{4\pi^2}$$

or $$l = \frac{9.8\,ms^{-2} \times 1s^2}{4 \times 3.14 \times 3.14} = 0.25\ m$$

Frequency $$f = \frac{1}{T} = \frac{1}{1s} = 1\ Hz$$

7.6 ENERGY CONSERVATION IN SHM

Let us consider the case of a vibrating mass-spring system. When the mass m is pulled slowly, the spring is stretched by an amount x_o against the elastic restoring force F. It is assumed that stretching is done slowly so that acceleration is zero. According to Hooke's law

$$F = kx_o$$

When displacement = 0 force = 0

When displacement = x_o force = kx_o

Average force $$F = \frac{0 + kx_o}{2} = \frac{1}{2}kx_o$$

Work done in displacing the mass m through x_o is

$$W = Fd = \frac{1}{2}k\,x_o \cdot x_o = \frac{1}{2}k\,x_o^2$$

This work appears as elastic potential energy of the spring. Hence

$$P.E. = \frac{1}{2}k\,x_o^2 \qquad \text{..........} \qquad (7.21)$$

The Eq. 7.21 gives the maximum P.E. at the extreme position. Thus

$$P.E._{max} = \frac{1}{2}k\,x_o^2$$

At any instant t, if the displacement is x, then P.E. at that instant is given by

$$P.E. = \frac{1}{2}kx^2 \quad \text{.........} \quad (7.22)$$

The velocity at that instant is given by Eq. 7.15 which is

$$v = x_o\sqrt{\frac{k}{m}\left(1-\frac{x^2}{{x_o}^2}\right)}$$

Hence the K.E. at that instant is

$$\text{K.E. of the mass} = \frac{1}{2}mv^2 = \frac{1}{2}m{x_o}^2\left(\frac{k}{m}\right)\left(1-\frac{x^2}{{x_o}^2}\right)$$

$$K.E. = \frac{1}{2}k{x_o}^2\left(1-\frac{x^2}{x_o^2}\right) \quad \text{.........} \quad (7.23)$$

Thus, kinetic energy is maximum when $x = 0$, i.e. when the mass is at equilibrium or mean position (Fig. 7.8)

$$K.E._{max} = \frac{1}{2}kx_o^2 \quad \text{.........} \quad (7.24)$$

For any displacement x, the energy is partly P.E. and partly K.E. Hence,

$$E_{total} = P.E. + K.E.$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}kx_o^2\left(1-\frac{x^2}{x_o^2}\right)$$

$$\text{Total energy} = \frac{1}{2}kx_o^2 \quad \text{.........} \quad (7.25)$$

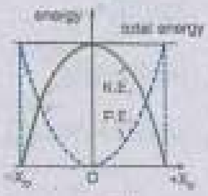


Fig. 7.8

Thus the total energy of the vibrating mass and spring is constant. When the K.E. of the mass is maximum, the P.E. of the spring is zero. Conversely, when the P.E. of the spring is maximum, the K.E. of the mass is zero. The interchange occurs continuously from one form to the other as the spring is compressed and released alternately. The variation of P.E. and K.E. with displacement is essential for maintaining oscillations. This periodic exchange of energy is a basic property of all oscillatory systems. In the case of simple pendulum gravitational P.E. of the mass, when displaced, is converted into K.E. at the equilibrium position. The K.E. is converted into P.E. as the mass rises to the top of the swing. Because of the frictional forces, energy is dissipated and consequently, the systems do not oscilate indefinitely.

Example 7.3: A spring, whose spring constant is $80.0\ Nm^{-1}$ vertically supports a mass of 1.0 kg in the rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with a velocity of $1.0\ ms^{-1}$.

Solution:

$k = 80.0\ Nm^{-1}$ $\qquad$ $m = 1.0\ kg$

Since $\omega^2 = \frac{k}{m}$ $\qquad$ or $\qquad$ $\omega = \sqrt{\frac{k}{m}}$

$$= \sqrt{\frac{80\ Nm^{-1}}{1kg}} = \sqrt{\frac{80\ kgms^{-2} \times m^{-1}}{1kg}} = 8.94\ s^{-1}$$

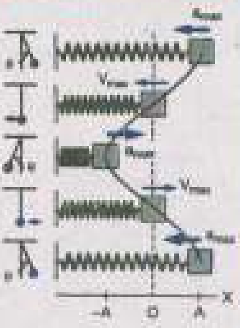
Let the amplitude of vibration be x_o

Then $\quad v = x_o\omega \qquad$ or $\qquad x_o = \frac{v}{\omega}$

as $\quad v = 1.0\ ms^{-1} \qquad$ and $\qquad \omega = 8.94\ s^{-1}$

Distance through which m is pulled $= x_o = \frac{1ms^{-1}}{8.94s^{-1}} = 0.11\ m$

Comparison of SHMs



7.7 FREE AND FORCED OSCILLATIONS

A body is said to be executing free vibrations when it oscillates without the interference of an external force. The frequency of these free vibrations is known as its natural frequency. For example, a simple pendulum when slightly displaced from its mean position vibrates freely with its natural frequency that depends only upon the length of the pendulum.

On the other hand, if a freely oscillating system is subjected to an external periodic force, then forced vibrations will take place. Such as when the mass of a vibrating pendulum is struck repeatedly, then forced vibrations are produced.

A physical system under going forced vibrations is known as driven harmonic oscillator.

The vibrations of a vehicle body caused by the running of engine is an example of forced vibrations. Another example of forced vibration is loud music produced by sounding wooden boards of string instruments.

7.8 RESONANCE

Associated with the motion of a driven harmonic oscillator, there is a very striking phenomenon, known as resonance. It arises if the external driving force is periodic with a period comparable to the natural period of the oscillator.

In a resonance situation, the driving force may be feeble, the amplitude of the motion may become extra ordinarily large. In the case of oscillating simple pendulum, if we blow to push the pendulum whenever it comes in front of our mouth, it is found that the amplitude steadily increases.

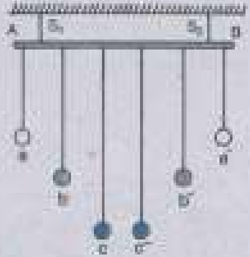


Fig. 7.9

To demonstrate this resonance effect, an apparatus is shown in Fig. 7.9. A horizontal rod AB is supported by two strings S_1 and S_2. Three pairs of pendulums aa', bb' and cc' are suspended to this rod. The length of each pair is the same but is different for different pairs. If one of these pendulums, say c, is displaced in a direction perpendicular to the plane of the paper, then its resultant oscillatory motion causes in rod AB a very slight disturbing motion, whose period is the same as that of c'. Due to this slight motion of the rod, each of the remaining pendulums (aa', bb', and cc') under go a slight periodic motion. This causes the pendulum c', whose length and, hence, period is exactly the same as that of c, to oscillate back and forth with steadily increasing amplitude. However, the amplitudes of the other pendulums remain small through out the subsequent motions of c and c' because their natural periods are not the same as that of the disturbing force due to rod AB.

The energy of the oscillation comes from the driving source. At resonance ,the transfer of energy is maximum.

Do You Know?

All structures are likely to resonate at one or more frequencies. This can cause problem. It is especially important to test all the components in helicopters and aeroplanes. Resonance in an aeroplane's wing or a helicopter rotor could be very dangerous.

Thus resonance occurs when the frequency of the applied periodic forced is equal to one of the natural frequencies of vibration of the forced or driven harmonic oscillator.

Interesting Information

The collapse of Tacoma Narrow bridge (USA) is suspected to be due to violent resonance oscillations.

Advantages And Disadvantages of Resonance

We come across many examples of resonance in every day life. A swing is a good example of mechanical resonance. It is like a pendulum with a single natural frequency depending on its length. If a series of regular pushes are given to the swing, its motion can be built up enormously. If pushes are given irregularly, the swing will hardly vibrate.

The column of soldiers, while marching on a bridge of long span are advised to break their steps. Their rhythmic march might set up oscillations of dangerously large amplitude in the bridge structure.

Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio, to tune a station, we are changing the natural frequency of the electric circuit of the receiver, to make it equal to the transmission frequency of the radio station. When the two frequencies match, energy absorption is maximum and this is the only station we hear.

Another good example of resonance is the heating and cooking of food very efficiently and evenly by microwave oven (Fig. 7.10). The waves produced in this type of oven have a wavelength of 12 cm at a frequency of 2450 MHz. At this frequency the waves are absorbed due to resonance by water and fat molecules in the food, heating them up and so cooking the food.

Fig. 7.10

7.9 DAMPED OSCILLATIONS

This is a common observation that the amplitude of an oscillating simple pendulum decreases gradually with time till it becomes zero. Such oscillations, in which the amplitude decreases steadily with time, are called damped oscillations.

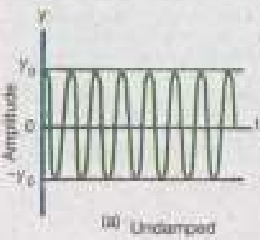


Fig. 7.11(a)
Graph between amplitude and time

We know from our everyday experience that the motion of any macroscopic system is accompanied by frictional effects. While describing the motion of a simple pendulum, this effect was completely ignored. As the bob of the pendulum moves to and fro, then in addition to the weight of the bob and the tension in the string, bob experiences viscous drag due to its motion through the air. Thus simple harmonic motion is an idealization (Fig. 7.11 a). In practice, the amplitude of this motion gradually becomes smaller and smaller because of friction and air resistance because the energy of the oscillator is used up in doing work against the resistive forces. Fig.7.11(b) shows how the amplitude of a damped simple harmonic wave changes with time as compared with an ideal un-damped harmonic wave. Thus we see that

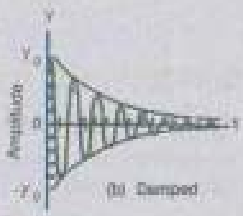


Fig. 7.11(b)
Graph between amplitude and time

Damping is the process whereby energy is dissipated from the oscillating system.

An application of damped oscillations is the shock absorber of a car which provides a damping force to prevent excessive oscillations (Fig. 7.12).

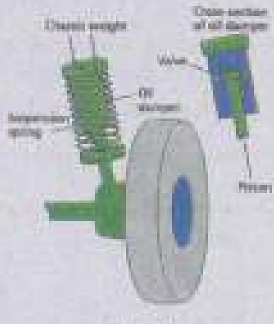


Fig. 7.12

7.10 SHARPNESS OF RESONANCE

We have seen that at resonance, the amplitude of the oscillator becomes very large. If the amplitude decreases rapidly at a frequency slightly different from the resonant frequency, the resonance will be sharp. The amplitude as well as its sharpness, both depend upon the damping. Smaller the damping, greater will be the amplitude and more sharp will be the resonance.

A heavily damped system has a fairly flat resonance curve as is shown in an amplitude frequency graph in Fig. 7.13.

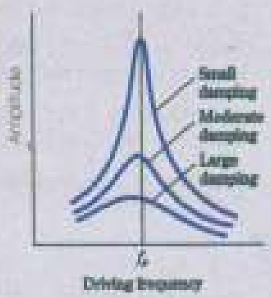


Fig. 7.13

The effect of damping can be observed by attaching a pendulum having light mass such as a pith ball as its bob and another of the same length carrying a heavy mass such as a lead bob of equal size, to a rod as shown in Fig. 7.9. They are set into vibrations by a third pendulum of equal length, attached to the same rod. It is observed that amplitude of the lead bob is much greater than that of the pith-ball. The damping effect for the pith-ball due to air resistance is much greater than for the lead bob.

SUMMARY

- Oscillatory motion is to and fro motion about a mean position.
- Periodic motion is the one that repeats itself after equal intervals of time.
- Restoring force opposes the change in shape or length of a body and is equal and opposite to applied force.
- A vibratory motion in which acceleration is directly proportional to displacement from mean position and is always directed towards the mean position is known as simple harmonic motion.
- The projection of a particle moving in a circle executes SHM. Its time period T is $\frac{2\pi}{\omega}$.
- Phase of vibration is the quantity which indicates the state of motion of a vibrating particle generally referred by the phase angle.
- The vibratory motion of a mass attached to an elastic spring is SHM and its time period is $T = 2\pi\sqrt{\frac{m}{k}}$
- The vibratory motion of the bob of simple pendulum is also SHM and its time period is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- In an oscillating system P.E. and K.E. interchange and total energy is conserved.
- A body is said to be executing free oscillation if it vibrates with its own natural frequency without the interference of an external force.
- When a freely oscillating system is subjected to an external periodic force, then forced vibrations take place.
- Resonance is the specific response of a system to a periodic force acting with the natural vibrating period of the system.
- Damping is the process whereby energy is dissipated from the oscillating system.

QUESTIONS

7.1 Name two characteristics of simple harmonic motion.

7.2 Does frequency depends on amplitude for harmonic oscillators?

7.3 Can we realize an ideal simple pendulum?

7.4 What is the total distance travelled by an object moving with SHM in a time equal to its period, if its amplitude is A?

7.5 What happens to the period of a simple pendulum if its length is doubled? What happens if the suspended mass is doubled?

7.6 Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.

7.7 What is meant by phase angle? Does it define angle between maximum displacement and the driving force?

7.8 Under what conditions does the addition of two simple harmonic motions produce a resultant, which is also simple harmonic?

7.9 Show that in SHM the acceleration is zero when the velocity is greatest and the velocity is zero when the acceleration is greatest?

7.10 In relation to SHM, explain the equations:

(i) $y = A \sin(\omega t + \varphi)$

(ii) $\mathbf{a} = -\omega^2 \mathbf{x}$

7.11 Explain the relation between total energy, potential energy and kinetic energy for a body oscillating with SHM.

7.12 Describe some common phenomena in which resonance plays an important role.

7.13 If a mass spring system is hung vertically and set into oscillations, why does the motion eventually stop?

NUMERICAL PROBLEMS

7.1 A 100.0 g body hung on a spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

(Ans:0.20 kg)

7.2 A load of 15.0g elongates a spring by 2.00 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be its (i) period (ii) spring constant (iii) maximum speed of its vibration.

[Ans: (i) 1.26s, (ii) 7.35 Nm^{-1}, (iii) 49.0 $cm\,s^{-1}$]

7.3 An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find

(i) Period

(ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

[Ans: (i) 1.3 s, (ii) 3.0 ms^{-2}, 1.4 ms^{-1}, 7.6 J, 1.44 J]

7.4 A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant $k = 1960\ Nm^{-1}$. Find the maximum distance through which the spring will be compressed.

(Ans: 0.18 m)

7.5 A simple pendulum is 50.0 cm long. What will be its frequency of vibration at a place where $g = 9.8\ ms^{-2}$?

(Ans: 0.70 Hz)

7.6 A block of mass 1.6 kg is attached to a spring with spring constant $1000\ Nm^{-1}$, as shown in Fig. 7.14. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, $x = 0$, if the surface is frictionless.

(Ans: $0.50\ ms^{-1}$)

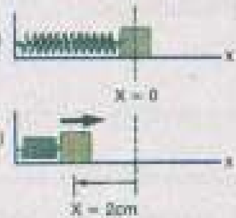


Fig. 7.14

7.7 A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant $20,000\ Nm^{-1}$. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

(Ans: 1.18 Hz)

7.8 Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation for its position, as a function of time, is

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t$$

What is the displacement of the object after 2.0 s?

(Ans: 0.25 m, $\frac{1}{16}$ Hz, 16 s, $x = 0.18$ m)

Chapter 8

WAVES

Learning Objectives

At the end of this chapter the students will be able to :

1. Recall the generation and propagation of waves.
2. Describe the nature of the motions in transverse and longitudinal waves.
3. Understand and use the terms wavelength, frequency and speed of wave.
4. Understand and use the equation $v = f\lambda$
5. Understand and describe Newton's formula of speed of sound.
6. Derive Laplace correction in Newton's formula of speed of sound for air.
7. Derive the formula $v = v_0 + 0.61\,t$
8. Recognize and describe the factors on which speed of sound in air depends.
9. Explain and use the principle of superposition.
10. Understand the terms interference and beats.
11. Describe the phenomena of interference and beats giving examples of sound waves.
12. Understand and describe reflection of waves.
13. Describe experiments, which demonstrate stationary waves for stretched strings and vibrating air columns.
14. Explain the formation of a stationary wave using graphical method.
15. Understand the terms node and anti-node.
16. Understand and describe modes of vibration of string.
17. Understand and describe Doppler's effect and its causes.
18. Recognize the applications of Doppler's effect in radar, sonar, astronomy, satellite and radar speed traps.

Waves transport energy without transporting matter. The energy transportation is carried by a disturbance, which spreads out from a source. We are well familiar with different types of waves such as water waves in the ocean, or gently formed ripples on a still pond due to rain drop. When a musician plucks a guitar-string, sound waves are generated which on reaching our ear, produce the sensation of music. Wave disturbances may also come in a concentrated bundle like the shock waves from an aeroplane flying at supersonic speed. Whatever may be the nature of waves, the mechanism by which it transports energy is the same. A succession of oscillatory motions are always involved. The wave is generated by an oscillation in the vibrating body and propagation of wave through space is by means of oscillations. The waves which propagate by the oscillation of material particles are known as mechanical waves.

Do You Know?

Ultrasonic waves are particularly useful for undersea communication and detection systems. High frequency radio waves, used in radars travel just a few centimetres in water, whereas highly directional beams of ultrasonic waves can be made to travel many kilometres.

There is another class of waves which, instead of material particles, propagate out in space due to oscillations of electric and magnetic fields. Such waves are known as electromagnetic waves. We will undertake the study of electromagnetic waves at a later stage. Here we will consider the mechanical waves only. The waves generated in ropes, strings, coil of springs, water and air are all mechanical waves.

So far we have been considering motion of individual particles but in case of mechanical waves, we study the collective motion of particles. An example will help us here. If you look at a black and white picture in a newspaper with a magnifying glass, you will discover that the picture is made up of many closely spaced dots. If you do not use the magnifier, you do not see the dots. What you see is the collective effect of dots in the form of a picture. Thus what we see as mechanical wave is actually the effect of oscillations of a very large number of particles of the medium through which the wave is passing.

8.1 PROGRESSIVE WAVES

Drop a pebble into water. Ripples will be produced and spread out across the water. The ripples are the examples of progressive waves because they carry energy across the water surface. A wave, which transfers energy by moving away from the source of disturbance, is called a progressive or travelling wave. There are two kinds of progressive waves - transverse waves and longitudinal waves.

Transverse and Longitudinal Waves

Consider two persons holding opposite ends of a rope or a hosepipe. Suddenly one person gives one up and down jerk to the rope. This disturbs the rope and creates a hump in it which travels along the rope towards the other person (Fig. 8.1 a & b).

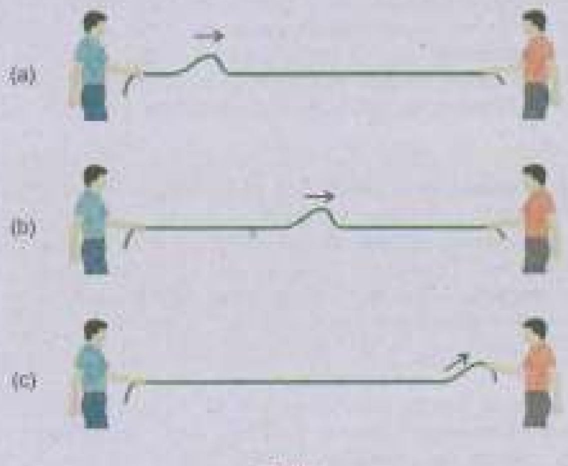


Fig. 8.1

When this hump reaches the other person, it causes his hand to move up (Fig. 8.1 c). Thus the energy and momentum imparted to the end of the rope by the first person has reached the other end of the rope by travelling through the rope i.e., a wave has been set up on the rope in the form of a moving hump. We call this type of wave a pulse. The forward motion of the pulse from one end of the rope to the other is an example of progressive wave. The hand jerking the end of the rope is the source of the wave. The rope is the medium in which the wave moves.

Fig. 8.2(a)

A large and loose spring coil (slinky spring) can be used to demonstrate the effect of the motion of the source in generating waves in a medium. It is better that the spring is laid on a smooth table with its one end fixed so that the spring does not sag under gravity.

If the free end of the spring is vibrated from side to side, a pulse of wave having a displacement pattern shown in Fig. 8.2 (a) will be generated which will move along the spring.

If the end of the spring is moved back and forth, along the direction of the spring itself as shown in Fig. 8.2 (b), a wave with back and forth displacement will travel along the spring. Waves like those in Fig. 8.2 (a) in which displacement of the spring is perpendicular to the direction of the waves are called transverse waves. Waves like those in Fig. 8.2 (b) in which displacements are in the direction of propagation of waves are called longitudinal waves. In this example the coil of spring is the medium, so in general we can say that

Fig. 8.2(b)

Transverse waves are those in which particles of the medium are displaced in a direction perpendicular to the direction of propagation of waves and longitudinal waves are those in which the particles of the medium have displacements along the direction of propagation of waves.

Both types of waves can be set up in solids. In fluids, however, transverse waves die out very quickly and usually cannot be produced at all. That is why, sound waves in air are longitudinal in nature.

8.2 PERIODIC WAVES

Fig. 8.3(a)

Upto now we have considered wave in the form of a pulse which is set up by a single disturbance in a medium like the snapping of one end of a rope or a coil spring. Continuous, regular and rhythmic disturbances in a medium result from periodic vibrations of a source which cause periodic waves in that medium. A good example of a periodic vibrator is an oscillating mass-spring system (Fig 8.3 a). We have already studied in the previous chapter that the mass of such a system executes SHM.

Transverse Periodic Waves

Imagine an experiment where one end of a rope is fastened to a mass spring vibrator. As the mass vibrates up and down, we observe a transverse periodic wave travelling along the length of rope (Fig. 8.3 b). The wave consists of crests and troughs. The crest is a pattern in which the rope is displaced above its equilibrium position, and in troughs, it has a displacement below its equilibrium position.

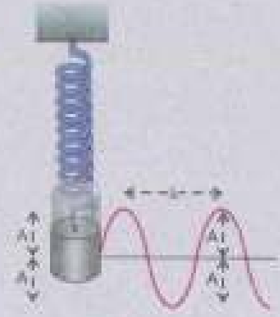


Fig. 8.3(b)

As the source executes harmonic motion up and down with amplitude A and frequency f, ideally every point along the length of the rope executes SHM in turn, with the same amplitude and frequency. The wave travels towards right as crests and troughs in turn, replace one another, but the points on the rope simply oscillates up and down. The amplitude of the wave is the maximum value of the displacement in a crest or trough and it is equal to the amplitude of the vibrator. The distance between any two consecutive crests or troughs is the same all along the length of the rope. This distance is called the wavelength of the periodic wave and is usually denoted by the Greek letter lambda λ (Fig. 8.3 b).

In principle, the speed of the wave can be measured by timing the motion of a wave crest over a measured distance. But it is not always convenient to observe the motion of the crest. As discussed below, however, the speed of a periodic wave can be found indirectly from its frequency and wavelength.

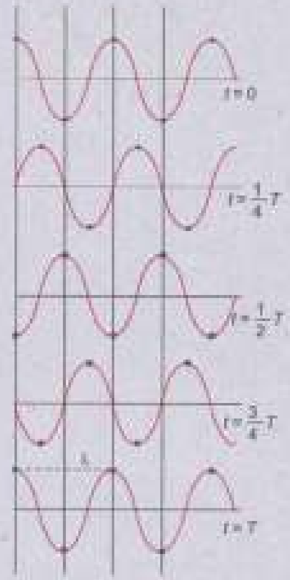


Fig. 8.4

As a wave progresses, each point in the medium oscillates periodically with the frequency and period of the source. Fig. 8.4 illustrates a periodic wave moving to the right, as it might look in photographic snapshots taken every ¼ period. Follow the progress of the crest that started out from the extreme left at $t = 0$. The time that this crest takes to move a distance of one wavelength is equal to the time required for a point in the medium to go through one complete oscillation. That is the crest moves one wavelength λ in one period of oscillation T. The speed v of the crest is therefore,

$$v = \frac{\text{distance moved}}{\text{corresponding time interval}} = \frac{\lambda}{T}$$

All parts of the wave pattern move with the same speed, so the speed of any one crest is just the speed of the wave. We can therefore, say that the speed v of the waves is

$$v = \frac{\lambda}{T} \quad \text{..........} \quad (8.1)$$

but $\frac{1}{T} = f$, where f is the frequency of the wave. It is the same as the frequency of the vibrator, generating the waves. Thus Eq. 8.1 becomes

$$v = f\lambda \quad \text{..........} \quad (8.2)$$

Phase Relationship between two Points on a Wave

Fig 8.5

The profile of periodic waves generated by a source executing SHM is represented by a sine curve. Figure 8.5 shows the snapshot of a periodic wave passing through a medium. In this figure, set of points are shown which are moving in unison as the periodic wave passes. The points C and C', as they move up and down, are always in the same state of vibration i.e., they always have identical displacements and velocities. Alternatively, we can say that as the wave passes, the points C ad C' move in phase. We may also say that C' leads C by one time period or 2π radian. Any point at a distance x, C lags behind by phase angle $\varphi = \frac{2\pi x}{\lambda}$

So is the case with points D and D'. Indeed there are infinitely many such points along the medium which are vibrating in phase. Points separated from one another through distances of λ, 2λ, 3λ, ... are all in phase with each other. These points can be anywhere along the wave and need not correspond with only the highest and lowest points. For example, points such as P, P', P'' .... are all in phase. Each is separated from the next by a distance λ.

Some of the points are exactly out of step. For example, when point C reaches its maximum upward displacement, at the same time D reaches its maximum downward displacement. At the instant that C begins to go down, D begins to move up. Points such as these are called one half period out of phase. Any two points separated from one another by $\frac{\lambda}{2}$, $3\frac{\lambda}{2}$, $5\frac{\lambda}{2}$, ... are out of phase.

Longitudinal Periodic Waves

In the previous section we have considered the generation of transverse periodic waves. Now we will see how the longitudinal periodic waves can be generated.

Consider a coil of spring as shown in Fig. 8.6. It is suspended by threads so that it can vibrate horizontally. Suppose an oscillating force F is applied to its end as indicated. The force will alternately stretch and compress the spring, thereby sending a series of stretched regions (called rarefaction) and compressions down the spring. We will see the oscillating force causes a longitudinal wave to move down the spring. This type of wave generated in springs is also called a compressional wave. Clearly in a compressional wave, the particles in the path of wave move back and forth along the line of propagation of the wave.

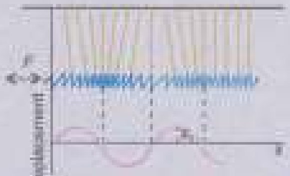


Fig. 8.6

Notice in Fig. 8.6, the supporting threads would be exactly vertical if the spring were undisturbed. The disturbance passing down the spring causes displacements of the elements of the spring from their equilibrium positions. In Fig. 8.6, the displacements of the thread from the vertical are a direct measure of the displacements of the spring elements. It is, therefore, an easy way to graph the displacements of the spring elements from their equilibrium positions and this is done in the lower part of the figure.

8.3 SPEED OF SOUND IN AIR

Sound waves are the most important examples of longitudinal or compressional waves. The speed of sound waves depends on the compressibility and inertia of the medium through which they are travelling. If the medium has the elastic modulus E and density ρ then, speed v is given by

$$v = \sqrt{\frac{E}{\rho}} \qquad \text{..........} \qquad (8.3)$$

Table 8.1

Speed of sound in different media

Medium	Speed $m\,s^{-1}$
Solids at 20°C	
Lead	1320
Copper	3600
Aluminium	5100
Iron	5130
Glass	5500
Liquids at 20°C	
Methanol	1120
Water	1483
Gases at S.T.P.	
Carbon dioxide	258
Oxygen	316
Air	332
Helium	972
Hydrogen	1286

As seen from the table 8.1, the speed of sound is much higher in solids than in gases. This makes sense because the molecules in a solid are closer than in a gas and hence, respond more quickly to a disturbance.

In general, sound travels more slowly in gases than in solids because gases are more compressible and hence have a smaller elastic modulus. For the calculation of elastic modulus for air, Newton assumed that when a sound wave travels through air, the temperature of the air during compression remains constant and pressure changes from P to $(P+\Delta P)$ and therefore, the volume changes from V to $(V - \Delta V)$. According to Boyle's law

$$PV = (P + \Delta P)(V - \Delta V) \quad .... \quad (8.4)$$

or $$PV = PV - P\Delta V + V\Delta P - \Delta P\Delta V$$

The product $\Delta P\,\Delta V$ is very small and can be neglected. So, the above equation becomes

$$P\Delta V = V\Delta P \qquad \text{or} \qquad P = \frac{\Delta P}{\Delta V} \times V = \frac{\Delta P}{\Delta V/V} = E$$

The expression $\left(\frac{\Delta P}{\Delta V/V}\right)$ is the elastic modulus E at constant temperature. So, substituting P for E in equation 8.3, we get Newton's formula for the speed of sound in air. Hence

$$v = \sqrt{\frac{P}{\rho}} \quad .... \quad (8.5)$$

For Your Information

Values of constant

Types of gas	γ
Monoatomic	1.67
Diatomic	1.40
Polyatomic	1.29

On substituting the values of atmospheric pressure and density of air at S.T.P. in equation 8.5, we find that the speed of sound waves in air comes out to be 280 ms^{-1}, whereas its experimental value is 332 ms^{-1}.

To account for this difference, Laplace pointed out that the compressions and rarefactions occur so rapidly that heat of compressions remains confined to the region where it is generated and does not have time to flow to the neighbouring cooler regions which have undergone an expansion. Hence the temperature of the medium does not remain constant. In such case Boyle's law takes the form

$$PV^{\gamma} = \text{Constant} \quad .... \quad (8.6)$$

where $$\gamma = \frac{\text{Molar specific heat of gas at constant pressure}}{\text{Molar specific heat of gas at constant volume}}$$

If the pressure of a given mass of a gas is changed from P to $(P + \Delta P)$ and volume changes from V to $(V - \Delta V)$, then using Eq. 8.6

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma}$$

$$PV^{\gamma} = (P + \Delta P)V^{\gamma}\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

Applying Binomial theorem

$$\left(1 - \frac{\Delta V}{V}\right)^{\gamma} = 1 - \gamma \frac{\Delta V}{V} + \text{negligible terms}$$

Hence $$P = (P + \Delta P)\left(1 - \gamma \frac{\Delta V}{V}\right)$$

or $$P = P - \gamma P \frac{\Delta V}{V} + \Delta P - \gamma \Delta P \frac{\Delta V}{V}$$

where $\left(\gamma \Delta P \frac{\Delta V}{V}\right)$ is negligible. Hence, we have

$$0 = -\gamma P \frac{\Delta V}{V} + \Delta P$$

or $$\frac{\Delta P}{\Delta V / V} = \gamma P = E$$

Thus elastic modulus $\left(\frac{\Delta P}{\Delta V / V}\right)$ equals γP.

Hence, substituting the value of elastic modulus in Eq. 8.3, we get Laplace expression for the speed of sound in a gas

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad \text{..........} \qquad (8.7)$$

For air $\gamma = 1.4$ so at S.T.P.

$$v = \sqrt{1.4} \times 280\ \text{ms}^{-1} = 333\ \text{ms}^{-1}$$

This value is very close to the experimental value.

For Your Information
Ranges of Hearing

Organisms	Frequencies (Hz)
Dolphin	150 - 150,000
Bat	1000 - 120,000
Cat	60 – 70,000
Dog	15 – 50,000
Human	20 - 20,000

Effect of Variation of Pressure, Density and Temperature on the Speed of Sound in a Gas

1. **Effect of Pressure:** Since density is proportional to the pressure, the speed of sound is not affected by a variation in the pressure of the gas.

2. **Effect of Density:** At the same temperature and pressure for the gases having the same value of γ, the speed is inversely proportional to the square root of their densities Eq. 8.7. Thus the speed of sound in hydrogen is four times its speed in oxygen as density of oxygen is 16 times that of hydrogen.

3. **Effect of Temperature:** When a gas is heated at constant pressure, its volume is increased and hence its density is decreased. As

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

So, the speed is increased with rise in temperature.

Let

v_0 = Speed of sound at 0 °C , ρ_0 = Density of gas at 0 °C

v_t = Speed of sound at t °C , ρ_t = Density of gas at t °C

then $$v_0 = \sqrt{\frac{\gamma P}{\rho_0}} \quad \text{and} \quad v_t = \sqrt{\frac{\gamma P}{\rho_t}}$$

Hence, $$\frac{v_t}{v_0} = \sqrt{\frac{\rho_0}{\rho_t}} \qquad \text{..........} \qquad (8.8)$$

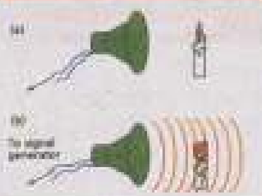
We have studied the volume expansion of gases in previous classes. If V_0 is the volume of a gas at temperature 0 °C and V_t is volume at t °C, then

$$V_t = V_0 (1 + \beta t)$$

Where β is the coefficient of volume expansion of the gas. For all gases, its value is about $\frac{1}{273}$. Hence

$$V_t = V_0 \left(1 + \frac{t}{273}\right)$$

Tidbits



Sound waves cause the candle flame to flicker.

Since $\quad \text{Volume} = \dfrac{\text{mass}}{\text{density}}$

Hence $$\frac{m}{\rho_t} = \frac{m}{\rho_o}\left(1 + \frac{t}{273}\right)$$

or $$\rho_o = \rho_t\left(1 + \frac{t}{273}\right)$$

Putting the value of ρ_o in equation 8.8 we have,

$$\frac{v_t}{v_o} = \sqrt{1 + \frac{t}{273}} \quad \text{.........} \quad (8.9)$$

or $$\frac{v_t}{v_o} = \sqrt{\frac{273 + t}{273}} = \sqrt{\frac{T}{T_o}} \quad \text{.........} \quad (8.10)$$

where T and T_o are the absolute temperatures corresponding to t °C and 0 °C respectively. Thus, the speed of sound varies directly as the square root of absolute temperature.

Expanding the R.H.S. of equation (8.9), using Binomial theorem and neglecting higher powers, we have

$$\frac{v_t}{v_o} = \left(1 + \frac{t}{546}\right) \quad \text{or} \quad v_t = v_o + \frac{v_o t}{548}$$

As $\quad v_o = 332\ \text{ms}^{-1}$

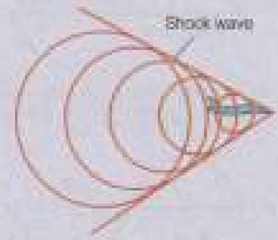
putting this value in the 2nd factor

Then $$v_t = v_o + \frac{332}{546}t$$

or $$v_t = v_o + 0.61\,t \quad \text{.........} \quad (8.11)$$

Do You Know?

Slower than the speed of sound.



Faster than the speed of sound.

What happens when a jet plane like Concorde flies faster than the speed of sound?
A conical surface of concentrated sound energy sweeps over the ground as a supersonic plane passes overhead. It is known as sonic boom.

Example 8.1: Find the temperature at which the velocity of sound in air is two times its velocity at 10 °C.

Solution: 10 °C = 10 °C + 273 = 283 K

Suppose at T K, the velocity is two times its value at 283 K.

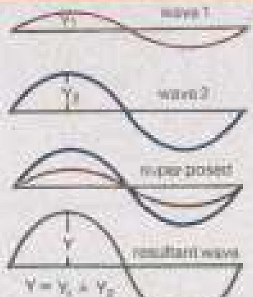
Fig 8.7

Since $\dfrac{v_T}{v_{283}} = \sqrt{\dfrac{T}{283\ K}}$

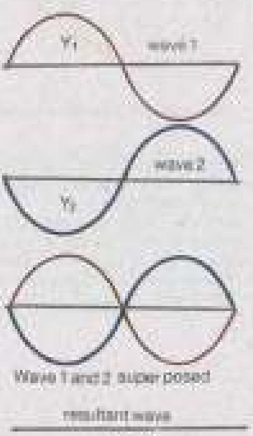
Therefore, $\dfrac{v_T}{v_{283}} = \sqrt{\dfrac{T}{283\ K}} = 2$

or $T = 1132\ K$ or $859\ ^{\circ}C$

For Your Information



Superposition of two waves of the same frequency which are exactly in phase.



Superposition of two waves of the same frequency which are exactly out of phase.

8.4 PRINCIPLE OF SUPERPOSITION

So far, we have considered single waves. What happens when two waves encounter each other in the same medium? Suppose two waves approach each other on a coil of spring, one travelling towards the right and the other travelling towards left. Fig. 8.7 shows what you would see happening on the spring. The waves pass through each other without being modified. After the encounter, each wave shape looks just as it did before and is travelling along just as it was before.

This phenomenon of passing through each other unchanged can be observed with all types of waves. You can easily see that it is true for surface ripples.

But what is going on during the time when the two waves overlap? Fig. 8.7 (c) shows that the displacements they produce just add up. At each instant, the spring's displacement at any point in the overlap region is just the sum of the displacements that would be caused by each of the two waves separately.

Thus, if a particle of a medium is simultaneously acted upon by n waves such that its displacement due to each of the individual n waves be y_1, y_2, ..., y_n, then the resultant displacement of the particle, under the simultaneous action of these n waves is the algebraic sum of all the displacements i.e.,

$$Y = y_1 + y_2 + \ldots\ldots + y_n$$

This is called principle of superposition.

Again, if two waves which cross each other have opposite phase, their resultant displacement will be

$$Y = y_1 - y_2$$

Particularly if $y_1 = y_2$ then result displacement Y= 0. Principle of superposition leads to many interesting phenomena with waves.

i) Two waves having same frequency and travelling in the same direction (Interference).

ii) Two waves of slightly different frequencies and travelling in the same direction (Beats)

iii) Two waves of equal frequency travelling in opposite direction (Stationary waves).

8.5 INTERFERENCE

Superposition of two waves having the same frequency and travelling in the same direction results in a phenomenon called interference.

An experimental set up to observe interference effect in sound waves is shown in Fig. 8.8 (a).

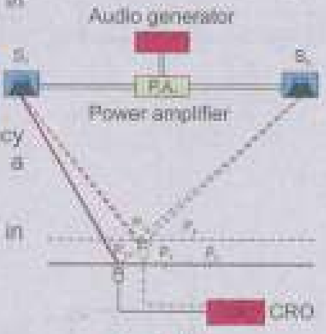


Fig. 8.8(a)

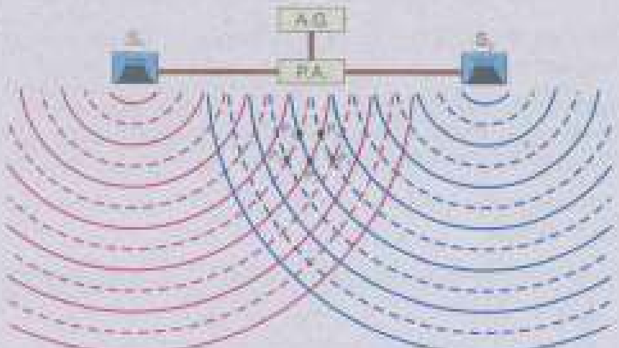


Fig. 8.8 (b)

Interference of sound waves
Points P_1, P_2, P_3 are points of constructive interference.
Points P_4 and P_5 are points of destructive interference.

Two loud speakers S_1 and S_2 act as two sources of harmonic sound waves of a fixed frequency produced by

An audio generator. Since the two speakers are driven from the same generator, they vibrate in phase. Such sources of waves are called coherent sources. A microphone attached to a sensitive cathode ray oscilloscope (CRO) acts as a detector of sound waves. The CRO is a device to display the input signal into waveform on its screen. The microphone is placed at various points, turn by turn, in front of the loud speakers as shown in the Fig. 8.8 (b).

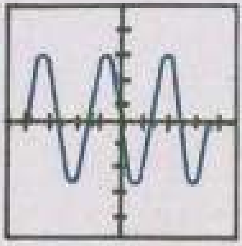


Fig. 8.8(c)
Constructive interference
Large displacement is displayed on the CRO screen

At points P_1, P_3 and P_5 a large signal is seen on the CRO [Fig. 8.8(c)], whereas at points P_2 and P_4 no signal is displayed on CRO screen [Fig. 8.8 (d)].This effect is explained in Fig. 8.8 (b) in which compressions and rarefactions are alternately emitted by both speakers. Continuous lines show compression and dotted lines show rarefactions. At points P_1, P_3 and P_5, we find that compression meets with a compression and rarefaction meets a rarefaction. So, the displacement of two waves are added up at these points and a large resultant displacement is produced which is seen on the CRO screen Fig. 8.8 (c).

Now from Fig. 8.8 (b), we find that the path difference ΔS between the waves at the point P_1 is

$$\Delta S = S_2P_1 - S_1P_1 \quad \text{or} \quad \Delta S = 4\tfrac{1}{2}\lambda - 3\tfrac{1}{2}\lambda = \lambda$$

Similarly at points P_3 and P_5, path difference is zero and $-\lambda$ respectively.

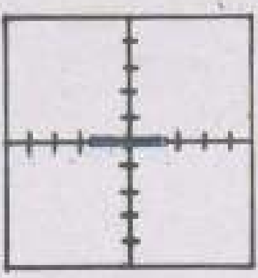


Fig. 8.8(d)
Destructive interference
Zero displacement is displayed on the CRO screen

Whenever path difference is an integral multiple of wavelength, the two waves are added up. This effect is called constructive interference.

Therefore, the condition for constructive interference can be written as

$$\Delta S = n\lambda \qquad \ldots\ldots\ldots \qquad (8.12)$$

where $n = 0, \pm1, \pm2, \pm3, \ldots\ldots\ldots$

At points P_2 and P_4, compression meets with a rarefaction, so that they cancel each other's effect. The resultant displacement becomes zero, as shown in [Fig. 8.8(d)].

Now let us calculate the path differene between the waves at points P_2 and P_4. For point P_2

$$\Delta S = S_2P_2 - S_1P_2 \quad \text{or} \quad \Delta S = 4\lambda - 3\frac{1}{2}\lambda = \frac{1}{2}\lambda$$

Similarly at P_4 the path difference is $-\frac{1}{2}\lambda$.

So, at points where the displacements of two waves cancel each other's effect, the path difference is an odd integral multiple of half the wavelength. This effect is called destructive interference.

Therefore, the condition for destructive interference can be written as

$$\Delta S = (2n + 1)\frac{\lambda}{2} \qquad \text{..........} \qquad (8.13)$$

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots\ldots$

8.6 BEATS

Tuning forks give out pure notes (single frequency). If two tuning forks A and B of the same frequency say 32Hz are sounded separately, they will give out pure notes. If they are sounded simultaneously, it will be difficult to differentiate the notes of one tuning fork from that of the other. The sound waves of the two will be superposed on each other and will be heard by the human ear as a single pure note. If the tuning fork B is loaded with some wax or plasticene, its frequency will be lowered slightly, say it becomes 30Hz.

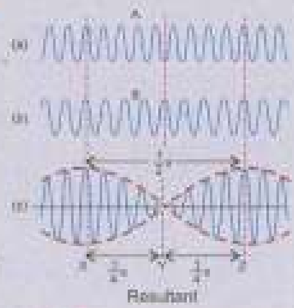


Fig. 8.9

Formation of Beats

If now the two tuning forks are sounded together, a note of alternately increasing and decreasing intensity will be heard. This note is called beat note or a beat which is due to interference between the sound waves from tuning forks A and B. Fig. 8.9 (a) shows the waveform of the note emitted from a tuning fork A. Similarly Fig. 8.9 (b) shows the waveform of the note emitted by tuning fork B. When both the tuning forks A and B are sounded together, the resultant waveform is shown in Fig. 8.9 (c).

Fig. 8.9 (c) shows how does the beat note occur. At some instant X the displacement of the two waves is in the same direction. The resultant displacement is large and a loud sound is heard.

After 1/4 s the displacement of the wave due to one tuning fork is opposite to the displacement of the wave due to the other tuning fork resulting in a minimum displacement at Y, hence, faint sound or no sound is heard.

Another 1/4 s later the displacements are again in the same direction and a loud sound is heard again at Z.

This means a loud sound is heard two times in each second. As the difference of the frequency of the two tuning forks is also 2 Hz so, we find that

Number of beats per second is equal to the difference between the frequencies of the tuning forks.

When the difference between the frequencies of the two sounds is more than about 10 Hz, then it becomes difficult to recognize the beats.

One can use beats to tune a string instrument, such as piano or violin, by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beats are heard.

Example 8.2: A tuning fork A produces 4 beats per second with another tuning fork B. It is found that by loading B with some wax, the beat frequency increases to 6 beats per second. If the frequency of A is 320 Hz, determine the frequency of B when loaded.

Solution: Since the beat frequency is 4, the frequency of B is either 320 + 4 = 324 Hz or 320 - 4 = 316 Hz. By loading B, its frequency will decrease. Thus if 324 Hz is the original frequency, the beat frequency will reduce. On the other hand, if it is 316 Hz, the beat frequency will increase which is the case. So, the original frequency of the tuning fork B is 316 Hz and when loaded, it is 316 - 2 = 314 Hz.

8.7 REFLECTION OF WAVES

In an extensive medium, a wave travels in all directions from its source with a velocity depending upon the properties of the medium. However, when the wave comes across the boundary of two media, a part of it is reflected back. The reflected wave has the same wavelength and frequency but its phase may change depending upon the nature of the boundary.

Now we will discuss two most common cases of reflection at the boundary. These cases will be explained with the help of waves travelling in slinky spring. (A slinky spring is a loose spring which has small initial length but a relatively large extended length).

One end of the slinky spring is tied to a rigid support on a smooth horizontal table. When a sharp jerk is given up to the free end of the slinky spring towards the side A, a displacement or a crest will travel from free end to the boundary (Fig. 8.10 a). It will exert a force on bound end towards the side A. Since this end is rigidly bound and acts as a denser medium, it will exert a reaction force on the spring in opposite direction. This force will produce displacement downwards B and a trough will travel backwards along the spring (Fig. 8.10 b).

Fig. 8.10

From the above discussion it can be concluded that whenever a transverse wave, travelling in a rarer medium, encounters a denser medium, it bounces back such that the direction of its displacement is reversed. An incident crest on reflection becomes a trough.

This experiment is repeated with a little variation by attaching one end of a light string to a slinky spring and the other end to the rigid support as shown in Fig. 8.11. If now the spring is given a sharp jerk towards A, a crest travels along the spring as shown in Fig. 8.11. When this crest reaches the spring-string boundary, it exert a force on the string towards the side A. Since the string has a small mass as compared to spring, it does not oppose the motion of the spring. The end of the spring, therefore, continues its displacement towards A. The spring behaves as if it has been plucked up. In other words a crest is again created at the boundary of the spring-string system, which travels backwards along the spring. From this it can be concluded that when a transverse wave travelling in a denser medium, is reflected from the boundary of a rarer medium, the direction of its displacement remains the same. An incident crest is reflected as a crest. We are already familiar with the fact that the direction of displacement is

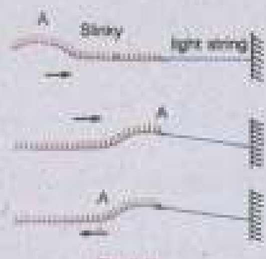


Fig. 8.11

reversed when there is change of 180° in the phase of vibration. So, the above conclusion can be written as follows.

i) If a transverse wave travelling in a rarer medium is incident on a denser medium, it is reflected such that it undergoes a phase change of 180°.

ii) If a transverse wave travelling in a denser medium is incident on a rarer medium, it is reflected without any change in phase.

8.8 STATIONARY WAVES

Now let us consider the superposition of two waves moving along a string in opposite directions. Fig. 8.12 (a,b) shows the profile of two such waves at instants $t = 0$, $T/4$, $3/4T$ and T, where T is the time period of the wave. We are interested in finding out the displacements of the points 1,2,3,4,5,6 and 7 at these instants as the waves superpose. From the Fig. 8.12 (a,b), it is obvious

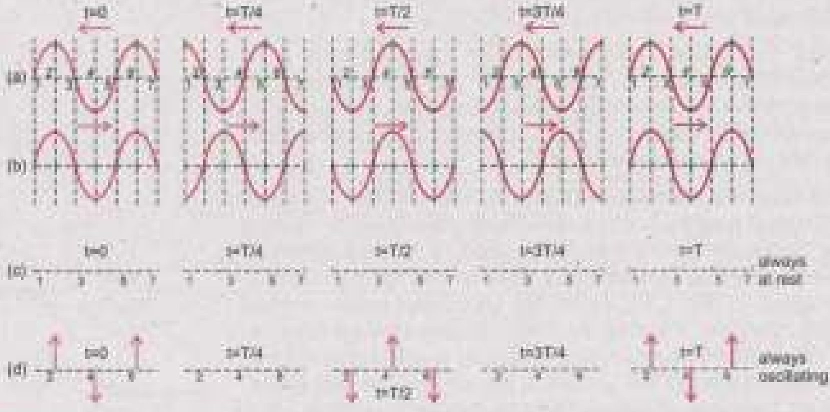


Fig 8.12

that the points 1,2,3, etc are distant $\lambda/4$ apart, λ being the wavelength of the waves. We can determine the resultant displacement of these points by applying the principle of superposition. Fig 8.12 (c) shows the resultant displacement of the points 1,3,5 and 7 at the instants $t = 0$, $T/4$, $T/2$, $3T/4$ and T. It can be seen that the resultant displacement of these points is always zero. These points of the medium are known as nodes. Fig. 8.12 (c) shows that the distance between two consecutive nodes is $\lambda/2$. Fig. 8.12 (d) shows the resultant displacement of the points 2,4 and 6 at the instants $t = 0$, $T/4$, $T/2$, $3T/4$ and T. The figure shows that these points are moving with an amplitude which is the sum of the amplitudes of the component waves. These points are known as antinodes. They are situated midway between the nodes and are also $\lambda/2$ apart. The distance between a node and the next antinode is $\lambda/4$. Such a pattern of nodes and anti-nodes is known as a stationary or standing wave.

Energy in a wave moves because of the motion of the particles of the medium. The nodes always remain at rest, so energy cannot flow past these points. Hence energy remains "standing" in the medium between nodes, although it alternates between potential and kinetic forms. When the antinodes are all at their extreme displacements, the energy stored is wholly potential and when they are simultaneously passing through their equilibrium positions, the energy is wholly kinetic.

An easy way to generate a stationary wave is to superpose a wave travelling down a string with its reflection travelling in opposite direction as explained in the next section.

8.9 STATIONARY WAVES IN A STRETCHED STRING

Consider a string of length l which is kept stretched by clamping its ends so that the tension in the string is F. If the string is plucked at its middle point, two transverse waves will originate from this point. One of them will move towards the left end of the string and the other towards the right end. When these waves reach the two clamped ends, they are reflected back thus giving rise to stationary waves. As the two ends of the string are clamped, no motion will take place there. So nodes will be formed at the two ends and one mode of vibration of the string will be as shown in Fig. 8.13 with the two ends as nodes with one antinode in between. Visually the string seems to vibrate in one loop. As the distance between two consecutive nodes is one half of the wavelength of the waves set up in the string, so in this mode of vibration, the length l of the string is

Fig. 8.13

$$l = \frac{\lambda_1}{2} \quad \text{or} \quad \lambda_1 = 2l \qquad \text{..........} \qquad (8.14)$$

where λ_1 is the wavelength of the waves set up in this mode.

The speed v of the waves in the string depends upon the tension F of the string and m, the mass per unit length of the string. It is given by $$v = \sqrt{\frac{F}{m}} \quad \text{.........} \quad (8.15)$$

Knowing the speed v and wavelength λ_1, the frequency f_1 of the waves is given by

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2l} \quad \text{.........} \quad (8.16)$$

Substituting the value of v $$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}} \quad \text{.........} \quad (8.17)$$

Thus in the first mode of vibration shown in Fig. 8.13, waves of frequency f_1 only will be set up in the given string.

$l = \lambda = \frac{2l}{2}$

Fig. 8.14

If the same string is plucked from one quarter of its length, again stationary waves will be set up with nodes and antinodes as shown in Fig. 8.14. Note that now the string vibrates in two loops. This particular configuration of nodes and antinodes has developed because the string was plucked from the position of an antinode. As the distance between two consecutive nodes is half the wavelength, so the Fig. 8.14 shows that the length l of string is equal to the wavelength of the waves set up in this mode. If λ_2 is the measure of wavelength of these waves, then,

$$\lambda_2 = l \quad \text{.........} \quad (8.18)$$

A comparison of this equation with Eq. 8.14 shows the wavelength in this case is half of that in the first case.

Eq. 8.16 shows that the speed of waves depends upon the tension and mass per unit length of the string. It is independent of the point from where the string is plucked to generate the waves. So the speed v of the waves will be same in two cases.

If f_2 is frequency of vibration of string in its second mode, then by Eq. 8.2

$$v = f_2 \times \lambda_2 = f_2\, l \qquad \text{or} \qquad f_2 = \frac{v}{l} \quad \text{..} \quad (8.19)$$

Comparing it with Eq. 8.16, we get

$$f_2 = 2f_1$$

Thus when the string vibrates in two loops, its frequency becomes double than when it vibrates in one loop.

Fig. 8.15

Similarly by plucking the string properly, it can be made to vibrate in 3 loops, with nodes and antinodes as shown in Fig. 8.15.

In this case the frequency of waves will be $f_3 = 3\,f_1$ and the wavelength will be equal to $2l/3$. Thus we can say that if the string is made to vibrate in n loops, the frequency of stationary waves set up on the string will be

$$f_n = n\,f_1 \qquad \text{..........} \qquad (8.20)$$

and the wavelength

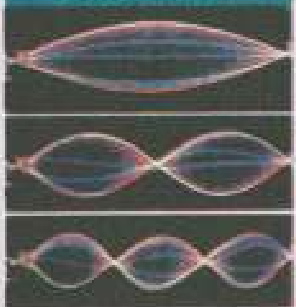
$$\lambda_n = \frac{2}{n}\,l \qquad \text{..........} \qquad (8.21)$$

It is clear that as the string vibrates in more and more loops, its frequency goes on increasing and the wavelength gets correspondingly shorter. However the product of the frequency and wavelength is always equal to v, the speed of waves.

The above discussion, clearly establishes that the stationary waves have a discrete set of frequencies f_1, $2f_1$, $3f_1$ ...., nf_1 which is known as harmonic series. The fundamental frequency f_1 corresponds to the first harmonic, the frequency $f_2 = 2\,f_1$ corresponds to the second harmonic and so on. The stationary waves can be set up on the string only with the frequencies of harmonic series determined by the tension, length and mass per unit length of the string. Waves not in harmonic series are quickly damped out.

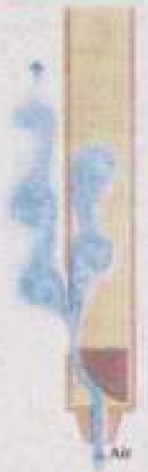
The frequency of a string on a musical instrument can be changed either by varying the tension or by changing the length. For example, the tension in guitar and violin strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck, thereby changing the length of the vibrating portion of the string.

Do You Know?



A standing wave pattern is formed when the length of the string is an integral multiple of half wave-length; otherwise no standing wave is formed.

For Your Information



In an organ pipe, the primary driving mechanism is wavering, sheet like jet of air from flute-slit, which interacts with the upper lip and the air column in the pipe to maintain a steady oscillation.

Example 8.3: A steel wire hangs vertically from a fixed point, supporting a weight of 80 N at its lower end. The diameter of the wire is 0.50 mm and its length from the fixed point to the weight is 1.5 m. Calculate the fundamental frequency emitted by the wire when it is plucked?

(Density of steel wire = 7.8×10^3 kgm^{-3})

Solution:

$$\text{Volume of wire} = \text{Length} \times \text{Area of cross section}$$

$$\text{Mass} = \text{Volume} \times \text{Density}$$

therefore

$$\text{Mass of wire} = \text{Length} \times \text{Area of cross section} \times \text{Density}$$

So, mass per unit length m is given by

$$m = \text{Density} \times \text{Area of cross section}$$

Diameter of the wire = D = 0.50 mm = 0.5×10^{-3} m

$$\text{Radius of the wire} = r = \frac{D}{2} = 0.25 \times 10^{-3}\ \text{m}$$

Area of cross section of wire = $\pi r^2 = 3.14 \times (0.25 \times 10^{-3}\ \text{m})^2$

$$F = w$$

therefore

$$m = 7.8 \times 10^3\ \text{kgm}^{-3} \times 3.14 \times (0.25 \times 10^{-3}\ \text{m})^2$$

$$m = 1.53 \times 10^{-3}\ \text{kgm}^{-1}$$

$$\text{Weight} = 80\ \text{N} = 80\ \text{kgms}^{-2}$$

Using the equation (8.17), we get

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}}$$

$$f_1 = \frac{1}{2 \times 1.5\ \text{m}}\sqrt{\frac{80\ \text{kgms}^{-2}}{1.53 \times 10^{-3}\ \text{kgm}^{-1}}} = 76\ \text{s}^{-1}$$

or

$$f_1 = 76\ \text{Hz}$$

8.10 STATIONARY WAVES IN AIR COLUMNS

Stationary waves can be set in other media also, such as air column. A common example of vibrating air column is in the organ pipe. The relationship between the incident wave and the reflected wave depends on whether the reflecting end of the pipe is open or closed. If the reflecting end is open, the air molecules have complete freedom of motion and this behaves as an antinode. If the reflecting end is closed, then it behaves as a node because the movement of the molecules is restricted. The modes of vibration of an air column in a pipe open at both ends are shown in Fig. 8.16.

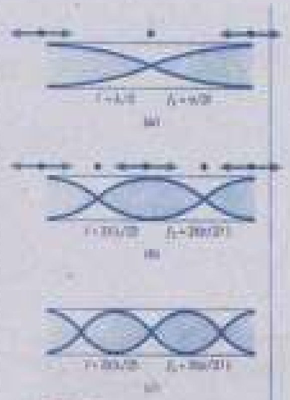


Fig. 8.16
Stationary longitudinal waves in a pipe open at both ends.

In figure, the longitudinal waves set up in the pipe have been represented by transverse curved lines indicating the varying amplitude of vibration of the air particles at points along the axis of the pipe. However, it must be kept in mind that air vibrations are longitudinal along the length of the pipe. The wavelength 'λ_n' of nth harmonic and its frequency 'f_n' of any harmonic is given by

$$\lambda_n = \frac{2l}{n} \quad , \quad f_n = \frac{v}{\lambda_n} = \frac{nv}{2l} \qquad \text{..........} \qquad (8.22)$$

$$n = 1, 2, 3, 4, \text{.........}$$

where 'v' is the speed of sound in air and 'l' is the length of the pipe. The equation 8.22 can also be written as

$$f_n = n f_1 \qquad \text{..........} \qquad (8.23)$$

If a pipe is closed at one end and open at the other, the closed end is a node. The modes of vibration in this case are shown in Fig. 8.17.

In case of fundamental note, the distance between a node and antinode is one fourth of the wavelength.

Hence, $\quad l = \dfrac{\lambda_1}{4} \quad$ or $\quad \lambda_1 = 4l$

Since $\quad v = f\lambda$

Hence $\quad f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{4l}$

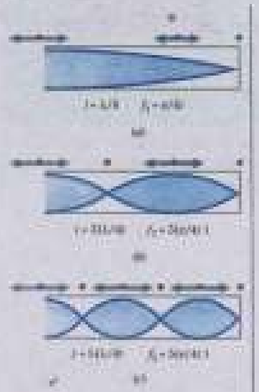


Fig. 8.17
Stationary longitudinal waves in a pipe closed at one end. Only odd harmonics are present.

It can be proved that in a pipe closed at one end, only odd harmonics are generated, which are given by the equation

$$f_n = \frac{nv}{4l} \quad \text{..........} \quad (8.24)$$

where $n = 1, 3, 5, \ldots\ldots\ldots$

This shows that the pipe, which is open at both ends, is richer in harmonics.

Example 8.4: A pipe has a length of 1 m. Determine the frequencies of the fundamental and the first two harmonics (a) if the pipe is open at both ends and (b) if the pipe is closed at one end.

(Speed of sound in air = 340 ms^{-1})

Solution:

a) $$f_1 = \frac{nv}{2l} = \frac{1 \times 340\, ms^{-1}}{2 \times 1\, m} = 170\, s^{-1} = 170\, Hz$$

$$f_2 = 2 f_1 = 2 \times 170\, Hz = 340\, Hz$$

and $$f_3 = 3 f_1 = 3 \times 170\, Hz = 510\, Hz$$

b) $$f_1 = \frac{nv}{4l} = \frac{1 \times 340\, ms^{-1}}{4 \times 1m} = 85\, s^{-1} = 85\, Hz$$

In this case only odd harmonics are present, so

$$f_3 = 3 f_1 = 3 \times 85\, Hz = 255\, Hz$$

and $$f_5 = 5 f_1 = 5 \times 85\, Hz = 425\, Hz$$

Interesting Information

Echolocation allows dolphins to detect small differences in the shape, size and thickness of objects.

8.11 DOPPLER EFFECT

An important phenomenon observed in waves is the Doppler effect. This effect shows that if there is some relative motion between the source of waves and the observer, an apparent change in frequency of the waves is observed.

This effect was observed by Johann Doppler while he was observing the frequency of light emitted from distant stars. In some cases, the frequency of light emitted from a star was found to be slightly different from that emitted from a similar source on the Earth. He found that the change in frequency of light depends on the motion of star relative to the Earth.

This effect can be observed with sound waves also. When an observer is standing on a railway platform, the pitch of the whistle of an approaching locomotive is heard to be higher. But when the same locomotive moves away, the pitch of the whistle becomes lower.

Fig. 8.18

An observer moving with velocity u_o towards a stationary source hears a frequency f_A that is greater than the source frequency.

The change in the frequency due to Doppler effect can be calculated easily if the relative motion between the source and the observer is along a straight line joining them. Suppose v is the velocity of the sound in the medium and the source emits a sound of frequency f and wavelength λ. If both the source and the observer are stationary, then the waves received by the observer in one second are $f = \frac{v}{\lambda}$. If an observer A moves towards the source with a velocity u_o (Fig. 8.18), the relative velocity of the waves and the observer is increased to $(v + u_o)$. Then the number of waves received in one second or modified frequency f_A is

$$f_A = \frac{v + u_o}{\lambda}$$

Putting the value of $\lambda = \frac{v}{f}$, the above equation becomes

$$f_A = f\left(\frac{v + u_o}{v}\right) \qquad \text{..........} \qquad (8.25)$$

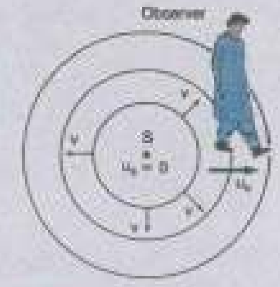


Fig. 8.19

An observer moving with velocity u_o away from stationary source hears a frequency f_B that is smaller than the source frequency.

For an observer B receding from the source (Fig. 8.19), the relative velocity of the waves and the observer is diminished to $(v - u_o)$. Thus the observer receives waves at a reduced rate. Hence, the number of waves received in one second in this case is $\left(\frac{v - u_o}{\lambda}\right)$

If the modified frequency, which the observer hears, is f_B then

$$f_B = \left(\frac{v - u_o}{\lambda}\right)$$

$$\text{or} \qquad f_B = \left(\frac{v - u_o}{v/f}\right) = f\left(\frac{v - u_o}{v}\right) \qquad \ldots\ldots\ldots\ldots \qquad (8.26)$$

Now, if the source is moving towards the observer with velocity u_s (Fig. 8.20), then in one second, the waves are compressed by an amount known as Doppler shift represented by $\Delta\lambda$.

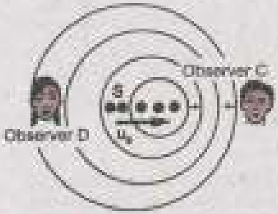


Fig. 8.20

A source moving with velocity u_s towards a stationary observer C and away from stationary observer D. Observer C hears an increased and observer D hears a decreased frequency.

$$\Delta\lambda = \left(\frac{u_s}{f}\right)$$

The compression of waves is due to the fact that same number of waves are contained in a shorter space depending upon the velocity of the source.

The wavelength for observer C is then

$$\lambda_C = \lambda - \Delta\lambda$$

$$\lambda_C = \left(\frac{v}{f} - \frac{u_s}{f}\right) = \left(\frac{v - u_s}{f}\right)$$

For the observer D, there will an increase in the wavelength given by:

$$\lambda_D = \lambda + \Delta\lambda$$

$$\lambda_D = \left(\frac{v}{f} + \frac{u_s}{f}\right) = \left(\frac{v + u_s}{f}\right)$$

The modified frequency for observer C is then

$$f_C = \frac{v}{\lambda_C} = \left(\frac{v}{v - u_s}\right)f \qquad \ldots\ldots\ldots\ldots \qquad (8.27)$$

and for the observer D will be

$$f_o = \frac{v}{\lambda_o} = \left(\frac{v}{v+u_s}\right)f \quad \text{..........} \quad (8.28)$$

This means that the observed frequency increases when the source is moving towards the observer and decreases when source is moving away from the observer.

Example 8.5: A train is approaching a station at 90 kmh^{-1} sounding a whistle of frequency 1000 Hz. What will be the apparent frequency of the whistle as heard by a listener sitting on the platform? What will be the apparent frequency heard by the same listener if the train moves away from the station with the same speed?

(speed of sound = 340ms^{-1})

Solution:

Frequency of source = f_o = 1000 Hz

Speed of sound = 340 ms^{-1}

Speed of train = u_s = 90 kmh^{-1} = 25 ms^{-1}

When train is approaching towards the listener, then using the relation

$$f' = \left(\frac{v}{v-u_s}\right)f$$

$$f' = \left(\frac{340\,\text{ms}^{-1}}{340\,\text{ms}^{-1} - 25\,\text{ms}^{-1}}\right) \times 1000\,\text{Hz} = 1079.4\,\text{Hz}$$

When train is moving away from the listener, then using the relation

$$f'' = \left(\frac{v}{v+u_s}\right)f$$

Do You Know?

The Doppler effect can be used to monitor blood flow through major arteries. Ultrasound waves of frequencies 5MHz to 10MHz are directed towards the artery and a receiver detects the back scattered signal. The apparent frequency depends on the velocity of flow of the blood.

$$f'' = \left(\frac{340\,ms^{-1}}{340\,ms^{-1} + 25\,ms^{-1}}\right) \times 1000\,Hz = 931.5\,Hz$$

Applications of Doppler Effect

Fig. 8.21

A frequency shift is used in a radar to detect the motion of an aeroplane

Doppler effect is also applicable to electromagnetic waves. One of its important applications is the radar system, which uses radio waves to determine the elevation and speed of an aeroplane. Radar is a device, which transmits and receives radio waves. If an aeroplane approaches towards the radar, then the wavelength of the wave reflected from aeroplane would be shorter and if it moves away, then the wavelength would be larger as shown in Fig. 8.21. Similarly speed of satellites moving around the Earth can also be determined by the same principle.

Sonar is an acronym derived from "Sound navigation and ranging". The general name for sonic or ultrasonic underwater echo-ranging and echo-sounding system. Sonar is the name of a technique for detecting the presence of objects underwater by acoustical echo.

In Sonar, "Doppler detection" relies upon the relative speed of the target and the detector to provide an indication of the target speed. It employs the Doppler effect, in which an apparent change in frequency occurs when the source and the observer are in relative motion to one another. Its known military applications include the detection and location of submarines, control of antisubmarine weapons, mine hunting and depth measurement of sea.

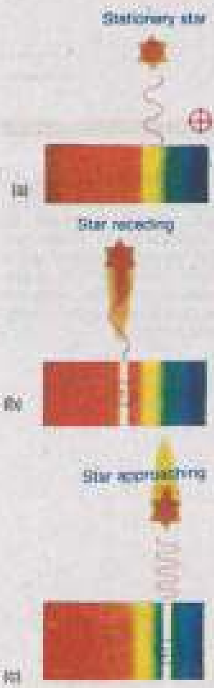


Fig. 8.22

Astronomers use the Doppler effect to calculate the speeds of distant stars and galaxies. By comparing the line spectrum of light from the star with light from a laboratory source, the Doppler shift of the star's light can be measured. Then the speed of the star can be calculated.

Stars moving towards the Earth show a blue shift. This is because the wavelength of light emitted by the star are shorter than if the star had been at rest. So, the spectrum is shifted towards shorter wavelength, i.e., to the blue end of the spectrum (Fig. 8.22).

Stars moving away from the Earth show a red shift. The emitted waves have a longer wavelength than if the star had been at rest. So the spectrum is shifted towards longer wavelength, i.e., towards the red end of the spectrum. Astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speeds.

Another important application of the Doppler shift using electromagnetic waves is the radar speed trap. Microwaves are emitted from a transmitter in short bursts. Each burst is reflected off by any car in the path of microwaves in between sending out bursts. The transmitter is opened to detect reflected microwaves. If the reflection is caused by a moving obstacle, the reflected microwaves are Doppler shifted. By measuring the Doppler shift, the speed at which the car moves is calculated by computer programme.

Do You Know?

Bats navigate and find food by echo location.

SUMMARY

- Waves carry energy and this energy is carried out by a disturbance, which spreads out from the source.
- If the particles of the medium vibrate perpendicular to the direction of propagation of the wave, then such wave is called transverse wave, e.g. light waves.
- If the particle of the medium vibrate parallel to the direction of propagation of the wave, then such wave is called longitudinal wave, e.g. sound waves.
- If a particle of the medium is simultaneously acted upon by two waves, then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called principle of superposition.
- When two waves meet each other in a medium then at some points they reinforce the effect of each other and at some other points they cancel each other's effect. This phenomenon is called interference.
- The periodic variations of sound between maximum and minimum loudness are called beats.
- Stationary waves are produced in a medium, when two identical waves travelling in opposite directions interfere in that medium.
- The apparent change in the pitch of sound caused by the relative motion of either the source of sound or the listener is called Doppler effect.

QUESTIONS

8.1 What features do longitudinal waves have in common with transverse waves?

8.2 The five possible waveforms obtained, when the output from a microphone is fed into the Y-input of cathode ray oscilloscope, with the time base on, are shown in Fig.8.23. These waveforms are obtained under the same adjustment of the cathode ray oscilloscope controls. Indicate the waveform

a) which trace represents the loudest note?

b) which trace represents the highest frequency?

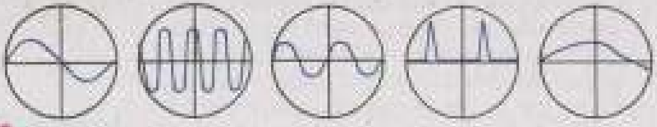


Fig. 8.23

8.3 Is it possible for two identical waves travelling in the same direction along a string to give rise to a stationary wave?

8.4 A wave is produced along a stretched string but some of its particles permanently show zero displacement. What type of wave is it?

8.5 Explain the terms crest, trough, node and antinode.

8.6 Why does sound travel faster in solids than in gases?

8.7 How are beats useful in tuning musical instruments?

8.8 When two notes of frequencies f_1 and f_2 are sounded together, beats are formed. If $f_1 > f_2$, what will be the frequency of beats?

i) $f_1 + f_2$ ii) $\frac{1}{2}(f_1 + f_2)$

iii) $f_1 - f_2$ iv) $\frac{1}{2}(f_1 - f_2)$

8.9 As a result of a distant explosion, an observer senses a ground tremor and then hears the explosion. Explain the time difference.

8.10 Explain why sound travels faster in warm air than in cold air.

8.11 How should a sound source move with respect to an observer so that the frequency of its sound does not change?

NUMERICAL PROBLEMS

8.1 The wavelength of the signals from a radio transmitter is 1500 m and the frequency is 200 kHz. What is the wavelength for a transmitter operating at 1000 kHz and with what speed the radio waves travel?

(Ans: 300 m, $3 \times 10^8\ ms^{-1}$)

8.2 Two speakers are arranged as shown in Fig. 8.24. The distance between them is 3 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.00 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the centre of the line and directly opposite each speakers. Calculate the speed of sound.

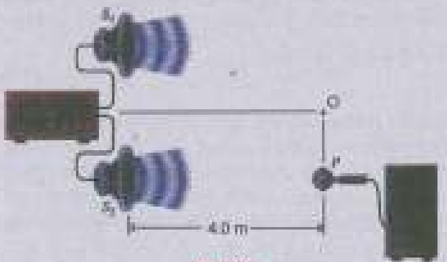


Fig. 8.24

(Ans: $344\ ms^{-1}$)

8.3 A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments, at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency?

(Ans: 0.6 m, 30 Hz)

8.4 The frequency of the note emitted by a stretched string is 300 Hz. What will be the frequency of this note when:

(a) the length of the wave is reduced by one-third without changing the tension.

(b) the tension is increased by one-third without changing the length of the wire.

(Ans: 450 Hz, 346 Hz)

8.5 An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is

(a) open at both ends.

(b) closed at one end.

(Speed of sound = $350\ ms^{-1}$)

[Ans: (a) 350 Hz, 700 Hz, (b) 175 Hz, 525 Hz]

8.6 A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes.

(Speed of sound = 340 ms^{-1})

(Ans: 21 Hz to 2833 Hz)

8.7 Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prong. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

(Ans: 253 Hz)

8.8 Two cars P and Q are travelling along a motorway in the same direction. The leading car P travels at a steady speed of 12 ms^{-1}; the other car Q, travelling at a steady speed of 20 ms^{-1}, sound its horn to emit a steady note which P's driver estimates, has a frequency of 830 Hz. What frequency does Q's own driver hear?

(Speed of sound = 340 ms^{-1})

(Ans: 810 Hz)

8.9 A train sounds its horn before it sets off from the station and an observer waiting on the plateform estimates its frequency at 1200 Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the plateform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50 s?

(Speed of sound = 340 ms^{-1})

(Ans: 17.9 ms^{-1}, 448 m)

8.10 The absorption spectrum of faint galaxy is measured and the wavelength of one of the lines identified as the Calcium α line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in a laboratory.

a) Is the galaxy moving towards or away from the Earth?

b) Calculate the speed of the galaxy relative to Earth.

(Speed of light = 3.0×10^8 ms^{-1})

[Ans: (a) away from the Earth, (b) 6.1×10^7 ms^{-1}]

Chapter 9

PHYSICAL OPTICS

Learning Objectives

At the end of this chapter the students will be able to:

1. Understand the concept of wavefront.
2. State Huygen's principle.
3. Use Huygen's principle to explain linear superposition of light.
4. Understand interference of light.
5. Describe Young's double slit experiment and the evidence it provided to support the wave theory of light.
6. Recognize and express colour patterns in thin films.
7. Describe the formation of Newton's rings.
8. Understand the working of Michelson's interferometer and its uses.
9. Explain the meaning of the term diffraction.
10. Describe diffraction at a single slit.
11. Derive the equation for angular position of first minimum.
12. Derive the equation $d \sin\theta = m\lambda$.
13. Carry out calculations using the diffraction grating formula.
14. Describe the phenomenon of diffraction of X-rays by crystals.
15. Appreciate the use of diffraction of X-rays by crystals.
16. Understand polarization as a phenomenon associated with transverse waves.
17. Recognize and express that polarization is produced by a Polaroid.
18. Understand the effect of rotation of Polaroid on polarization.
19. Understand how plane polarized light is produced and detected.

Light is a type of energy which produces sensation of vision. But how does this energy propagate? In 1678, Huygen's, an eminent Dutch scientist, proposed that

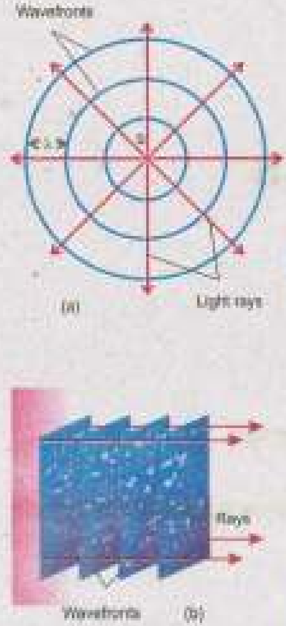
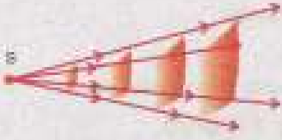


Fig. 9.1

Spherical wave fronts (a) and plane wavefronts(b)spaced a wavelength apart. The arrows represent rays.



Do You Know?

Small segments of large spherical wavefronts approximate a plane wavefront.

light energy from a luminous source travels in space as waves. The experimental evidence in support of wave theory in Huygen's time was not convincing. However, Young's interference experiment performed for the first time in 1801 proved wave nature of light and thus established the Huygen's wave theory. In this chapter you will study the properties of light, associated with its wave nature.

9.1 WAVEFRONTS

Consider a point source of light at S (Fig. 9.1 a). Waves emitted from this source will propagate outwards in all directions with speed c. After time t, they will reach the surface of an imaginary sphere with centre as S and radius as ct. Every point on the surface of this sphere will be set into vibration by the waves reaching there. As the distance of all these points from the source is the same, their state of vibration will be identical. In other words, all the points on the surface of the sphere will have the same phase.

Such a surface on which all the points have the same phase of vibration is known as wavefront.

Thus in case of a point source, the wavefront is spherical in shape. A line normal to the wavefront, showing the direction of propagation of light is called a ray of light.

With time, the wave moves farther giving rise to new wavefronts. All these wavefronts will be concentric spheres of increasing radii as shown in Fig. 9.1 (a). Thus the wave propagates in space by the motion of the wavefronts. The distance between the consecutive wavefronts is one wavelength. It can be seen that as we move away at greater distance from the source, the wavefronts are parts of spheres of very large radii. A limited region taken on such a wavefront can be regarded as a plane wavefront (Fig.9.1b). For example, light from the Sun reaches the Earth with plane wavefronts.

In the study of interference and diffraction, plane waves and plane wavefronts are considered. A usual way to obtain a plane wave is to place a point source of light at the focus of a convex lens. The rays coming out of the lens will constitute plane waves.

9.2 HUYGEN'S PRINCIPLE

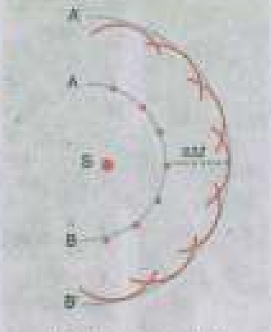
Knowing the shape and location of a wavefront at any instant t, Huygen's principle enables us to determine the shape and location of the new wavefront at a later time $t + \Delta t$. This principle consists of two parts:

(i) Every point of a wavefront may be considered as a source of secondary wavelets which spread out in forward direction with a speed equal to the speed of propagation of the wave.

(ii) The new position of the wavefront after a certain interval of time can be found by constructing a surface that touches all the secondary wavelets.

The principle is illustrated in Fig. 9.2 (a). AB represents the wavefront at any instant t. To determine the wavefront at time $t + \Delta t$, draw secondary wavelets with centre at various points on the wavefront AB and radius as $c\Delta t$ where c is speed of the propagation of the wave as shown in Fig.9.2 (a). The new wavefront at time $t + \Delta t$ is A'B' which is a tangent envelope to all the secondary wavelets.

Figure 9.2 (b) shows a similar construction for a plane wavefront.



(a) Spherical wavefront

(b) Plane wavefront

Fig.9.2

Huygens' construction for determining the position of the wavefronts AB and CD after a time interval Δt. A'B' and C'D' are the new positions of the wavefronts.

9.3 INTERFERENCE OF LIGHT WAVES

An oil film floating on water surface exhibits beautiful colour patterns. This happens due to interference of light waves, the phenomenon, which is being discussed in this section.

Conditions for Detectable Interference

It was studied in Chapter 8 that when two waves travel in the same medium, they would interfere constructively or destructively. The amplitude of the resultant wave will be greater then either of the individual waves, if they interfere constructively. In the case of destructive interference, the

For Your Information

Monochromatic Light
Sodium chloride in a flame gives out pure yellow light. This light is not a mixture of red and green.

amplitude of the resultant wave will be less than either of the individual waves.

Interference of light waves is not easy to observe because of the random emission of light from a source. The following conditions must be met, in order to observe the phenomenon.

1. The interfering beams must be monochromatic, that is, of a single wavelength.
2. The interfering beams of light must be coherent.

Consider two or more sources of light waves of the same wavelength. If the sources send out crests or troughs at the same instant, the individual waves maintain a constant phase difference with one another. The monochromatic sources of light which emit waves, having a constant phase difference, are called coherent sources.

A common method of producing two coherent light beams is to use a monochromatic source to illuminate a screen containing two small holes, usually in the shape of slits. The light emerging from the two slits is coherent because a single source produces the original beam and two slits serve only to split it into two parts. The points on a Huygen's wavefront which send out secondary wavelets are also coherent sources of light.

9.4 YOUNG'S DOUBLE SLIT EXPERIMENT

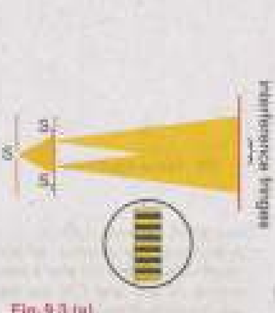


Fig. 9.3 (a)
Ray geometry of Young's double slit experiment.

Fig. 9.3 (a) shows the experimental arrangement, similar to that devised by Young in 1801, for studying interference effects of light. A screen having two narrow slits is illuminated by a beam of monochromatic light. The portion of the wavefront incident on the slits behaves as a source of secondary wavelets (Huygen's principle). The secondary wavelets leaving the slits are coherent. Superposition of these wavelets result in a series of bright and dark bands (fringes) which are observed on a second screen placed at some distance parallel to the first screen.

Let us now consider the formation of bright and dark bands. As pointed out earlier the two slits behave as coherent sources of secondary wavelets. The wavelets arrive at the screen in such a way that at some points crests fall on crests and troughs on troughs resulting in constructive interference and bright fringes are formed. There are some points on the screen where crests meet troughs giving rise to destructive interference and dark fringes are thus formed.

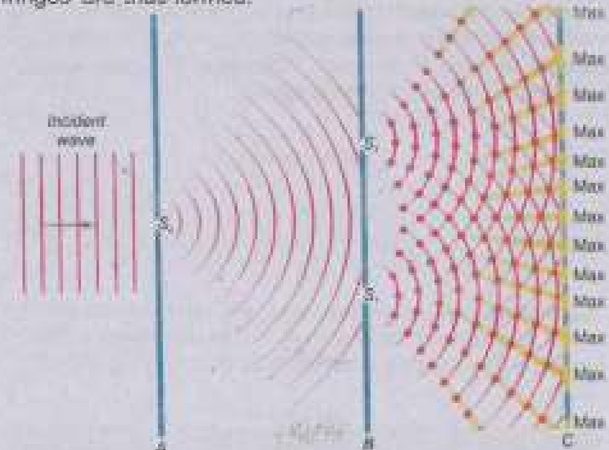


Fig. 9.3 (b)

Young's double slit experiment for interference of light.

The bright fringes are termed as maxima and dark fringes as minima.

In order to derive equations for maxima and minima, an arbitrary point P is taken on the screen on one side of the central point O as shown in Fig. 9.3 (c). AP and BP are the paths of the rays reaching P. The line AD is drawn such that AP = DP. The separation between the centres of the two slits is AB = d. The distance of the screen from the slits is CO = L. The angle between CP and CO is θ. It can be proved that the angle BAD = θ by assuming that AD is nearly normal to BP. The path difference between the wavelets, leaving the slits and arriving at P, is BD. It is the number of wavelengths, contained within BD, that determines whether bright or dark fringe will appear at P. If the point P is to have bright fringe, the path difference BD must be an integral multiple of wavelength.

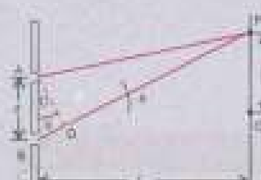


Fig. 9.3 (c)

Geometrical construction of Young's double slit experiment

Thus, BD = $m\lambda$, where $m = 0, 1, 2, \ldots\ldots$

Since $BD = d \sin\theta$

therefore $$d \sin\theta = m\lambda \quad \ldots\ldots\ldots \quad (9.1)$$

It is observed that each bright fringe on one side of O has symmetrically located bright fringe on the other side of O. The central bright fringe is obtained when $m = 0$. If a dark fringe appears at point P, the path difference BD must contain half-integral number of wavelengths.

Thus $$BD = \left(m + \frac{1}{2}\right)\lambda$$

therefore $$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad \ldots\ldots\ldots \quad (9.2)$$

The first dark fringe, in this case, will obviously appear for $m = 0$ and second dark for $m = 1$. The interference pattern formed in the Young's experiment is shown in Fig. 9.3 (d).

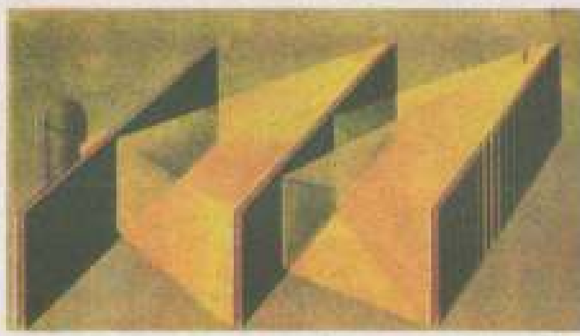


Fig. 9.3 (d)

An interference pattern by monochromatic light in Young's double slits experiment

For Your Information

θ	$\sin\theta$	$\tan\theta$
2	0.035	0.035
4	0.070	0.070
6	0.104	0.105
8	0.139	0.140
10	0.174	0.176

Equations 9.1 and 9.2 can be applied for determining the linear distance on the screen between adjacent bright or dark fringes. If the angle θ is small, then

$$\sin\theta \approx \tan\theta$$

Now from Fig. 9.3 (c), $\tan\theta = y/L$, where y is the distance of the point P from O. If a bright fringe is observed at P, then, from Eq. 9.1, we get,

$$y = m\frac{\lambda L}{d} \qquad \ldots\ldots\ldots \qquad (9.3)$$

If P is to have dark fringe it can be proved that

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \qquad \ldots\ldots\ldots \qquad (9.4)$$

In order to determine the distance between two adjacent bright fringes on the screen, mth and (m + 1) th fringes are considered.

For the mth bright fringe, $\qquad y_m = m\frac{\lambda L}{d}$

and for the (m + 1)th bright fringe $\qquad y_{m+1} = (m+1)\frac{\lambda L}{d}$

If the distance between the adjacent bright fringes is Δy, then

$$\Delta y = y_{m+1} - y_m = (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$$

Therefore, $$\Delta y = \frac{\lambda L}{d} \qquad \ldots\ldots\ldots \qquad (9.5)$$

Similarly, the distance between two adjacent dark fringes can be proved to be $\lambda L/d$. It is, therefore, found that the bright and dark fringes are of equal width and are equally spaced.

Eq. 9.5 reveals that fringe spacing increases if red light (long wavelength) is used as compared to blue light (short wavelength). The fringe spacing varies directly with distance L between the slits and screen and inversely with the separation d of the slits.

If the separation d between the two slits, the order m of a bright or dark fringe and fringe spacing Δy are known, the wavelength λ of the light used for interference effect can be determined by applying Eq. 9.5.

Tidbits

An interference pattern formed with white light.

Example 9.1: The distance between the slits in Young's double slit experiment is 0.25 cm. Interference fringes are formed on a screen placed at a distance of 100 cm from the slits. The distance of the third dark fringe from the central bright fringe is 0.059 cm. Find the wavelength of the incident light.

Solution: Given that

$$d = 0.25\,cm = 2.5 \times 10^{-3}\,m$$

$$y = 0.059\,cm = 5.9 \times 10^{-4}\,m$$

$$L = 100\,cm = 1\,m$$

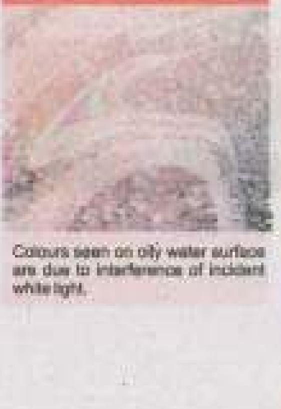
For the 3rd dark fringe m = 2

Using $$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d}$$

$$\lambda = \frac{5.9 \times 10^{-4}\,m \times 2.5 \times 10^{-3}\,m}{\left(2 + \frac{1}{2}\right) \times 1m}$$

Therefore, $$\lambda = 5.90 \times 10^{-7}\,m = 590\,nm$$

Interesting Information



Colours seen on oily water surface are due to interference of incident white light.

Example 9.2: Yellow sodium light of wavelength 589 nm, emitted by a single source passes through two narrow slits 1.00 mm apart. The interference pattern is observed on a screen 225 cm away. How far apart are two adjacent bright fringes?

Solution: Given that

$$\lambda = 589\,nm = 589 \times 10^{-9}\,m$$

$$d = 1.00\,mm = 1.00 \times 10^{-3}\,m$$

$$L = 225\,cm = 2.25\,m$$

$$\Delta y = ?$$

Using $$\Delta y = \frac{\lambda L}{d}$$

$$\Delta y = \frac{589 \times 10^{-9}\,m \times 2.25\,m}{1.0 \times 10^{-3}\,m}$$

$$\Delta y = 1.33 \times 10^{-3}\,m \quad \text{or} \quad 1.33\,mm.$$

Thus, the adjacent frings will be 1.33 mm apart.

9.5 INTERFERENCE IN THIN FILMS

A thin film is a transparent medium whose thickness is comparable with the wavelength of light. Brilliant and beautiful colours in soap bubbles and oil film on the surface of water are due to interference of light reflected from the two surfaces of the film as explained below.

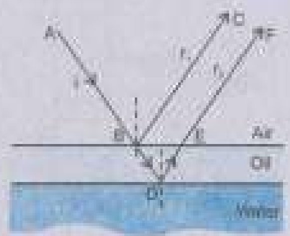


Fig. 9.4
Geometrical construction of interference of light due to a thin oil film

Consider a thin film of a refracting medium. A beam AB of monochromatic light of wavelength λ is incident on its upper surface. It is partly reflected along BC and partly refracted into the medium along BD. At D it is again partly reflected inside the medium along DE and then at E refracted along EF as shown in Fig. 9.4. The beams BC and EF, being the parts of the same primary beam have a phase coherence. As the film is thin, so the separation between the beams BC and EF will be very small, and they will superpose and the result of their interference will be detected by the eye. It can be seen in Fig. 9.4, that the original beam splits into two parts BC and EF due to the thin film enter the eye after covering different lengths of paths. Their path difference depends upon (i) thickness and nature of the film and (ii) angle of incidence. If the two reflected waves reinforce each other, then the film as seen with the help of a parallel beam of monochromatic light will look bright. However, if the thickness of the film and angle of incidence are such that the two reflected waves cancel each other, the film will look dark.

Do You Know?

The vivid iridescence of peacock feathers due to interference of the light reflected from its complex layered surface.

If white light is incident on a film of irregular thickness at all possible angles, we should consider the interference pattern due to each spectral colour separately. It is quite possible that at a certain place on the film, its thickness and the angle of incidence of light are such that the condition of destructive interference of one colour is being satisfied. Hence, that portion of the film will exhibit the remaining constituent colours of the white light as shown in Fig. 9.5.

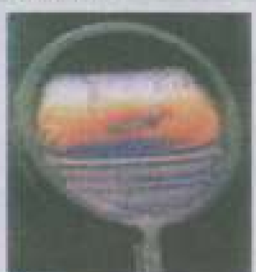


Fig. 9.5
Interference pattern produced by a thin soap film illuminated by white light.

9.6 NEWTON'S RINGS

When a plano-convex lens of long focal length is placed in contact with a plane glass plate (Fig. 9.6 a), a thin air film is enclosed between the upper surface of the glass plate and the lower surface of the lens. The thickness of the air film is

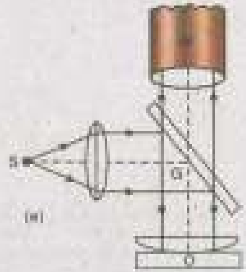


Fig. 9.6 (a)
Experimental arrangement for observing Newton's rings.

almost zero at the point of contact O and it gradually increases as one proceeds towards the periphery of the lens. Thus, the points where the thickness of air film is constant, will lie on a circle with O as centre.

By means of a sheet of glass G, a parallel beam of monochromatic light is reflected towards the plano-convex lens L. Any ray of monochromatic light that strikes the upper surface of the air film nearly along normal is partly reflected and partly refracted. The ray refracted in the air film is also reflected partly at the lower surface of the film. The two reflected rays, i.e. produced at the upper and lower surfaces of the film, are coherent and interfere constructively or destructively. When the light reflected upwards is observed through a microscope M which is focussed on the glass plate, series of dark and bright rings are seen with centre at O (Fig. 9.6 b). These concentric rings are known as Newton's rings.

Fig. 9.6 (b)
A pattern of Newton's rings due to interference of monochromatic light.

At the point of contact of the lens and the glass plate, the thickness of the film is effectively zero but due to reflection at the lower surface of air film from denser medium, an additional path difference of $\lambda/2$ is introduced. Consequently, the centre of Newton rings is dark due to destructive interference.

9.7 MICHELSON'S INTERFEROMETER

Michelson's interferometer is an instrument that can be used to measure distance with extremely high precision. Albert A. Michelson devised this instrument in 1881 using the idea of interference of light rays. The essential features of a Michelson's interferometer are shown schematically in Fig.9.7.

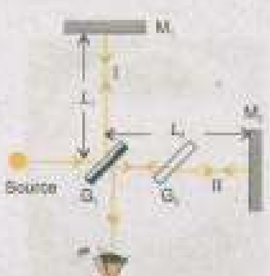


Fig 9.7
Schematic diagram of a Michelson's interferometer.

Monochromatic light from an extended source falls on a half silvered glass plate G_1 that partially reflects it and partially transmits it. The reflected portion labelled as I in the figure travels a distance L_1 to mirror M_1, which reflects the beam back towards G_1. The half silvered plate G_1 partially transmits this portion that finally arrives at the observer's eye. The transmitted portion of the original beam labelled as II, travels a distance L_2 to mirror M_2 which reflects the beam back toward G_1. The beam II partially reflected by G_1 also arrives the observer's eye finally. The plate G_2, cut from the same piece of glass as G_1, is introduced in the path of beam II as a compensator plate. G_2, therefore, equalizes the path length of the beams I and II in glass. The two beams having their different paths are coherent. They produce interference effects when they arrive at observer's eyes. The observer then sees a series of parallel interference fringes.

For Your Information

A photograph of Michelson interferometer.

In a practical interferometer, the mirror M_1 can be moved along the direction perpendicular to its surface by means of a precision screw. As the length L_1 is changed, the pattern of interference fringes is observed to shift. If M_1 is displaced through a distance equal to $\lambda/2$, a path difference of double of this displacement is produced, i.e., equal to λ. Thus a fringe is seen shifted forward across the line of reference of cross wire in the eye piece of the telescope used to view the fringes.

A fringe is shifted, each time the mirror is displaced through $\lambda/2$. Hence, by counting the number m of the fringes which are shifted by the displacement L of the mirror, we can write the equation,

$$L = m\frac{\lambda}{2} \qquad \text{..........} \qquad (9.6)$$

For Your Information

Interference fringes in the Michelson interferometer

Very precise length measurements can be made with an interferometer. The motion of mirror M_1 by only $\lambda/4$ produces a clear difference between brightness and darkness. For $\lambda = 400$ nm, this means a high precision of 100 nm or 10^{-4} mm.

Michelson measured the length of standard metre in terms of the wavelength of red cadmium light and showed that the standard metre was equivalent to 1,553,163.5 wavelengths of this light.

9.8 DIFFRACTION OF LIGHT

In the interference pattern obtained with Young's double slit experiment (Fig. 9.3 b), the central region of the fringe system is bright. If light travels in a straight line, the central region should appear dark i.e., the shadow of the screen between the two slits. Another simple experiment can be performed for exhibiting the same effect.

Consider that a small and smooth steel ball of about 3 mm in diameter is illuminated by a point source of light. The shadow of the object is received on a screen as shown in Fig. 9.8. The shadow of the spherical object is not completely dark but has a bright spot at its centre. According to Huygen's principle, each point on the rim of the sphere behaves as a source of secondary wavelets which illuminate the central region of the shadow.

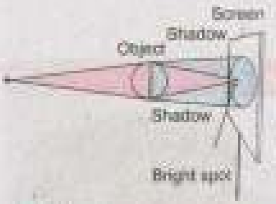


Fig.9.8

Bending of light caused by its passage past a spherical object.

These two experiments clearly show that when light travels past an obstacle, it does not proceed exactly along a straight path, but bends around the obstacle.

The property of bending of light around obstacles and spreading of light waves into the geometrical shadow of an obstacle is called diffraction.

Point to ponder

Hold two fingers close together to form a slit. Look at a light bulb through the slit. Observe the pattern of light being seen and think why it is so.

The phenomenon is found to be prominent when the wavelength of light is large as compared with the size of the obstacle or aperture of the slit. The diffraction of light occurs, in effect, due to the interference between rays coming from different parts of the same wavefront.

9.9 DIFFRACTION DUE TO A NARROW SLIT

Fig. 9.9 shows the experimental arrangement for studying diffraction of light due to a narrow slit. The slit AB of width d is illuminated by a parallel beam of monochromatic light of wavelength λ. The screen S is placed parallel to the slit for observing the effects of the diffraction of light. A small portion of the incident wavefront passes through the narrow slit. Each point of this section of the wavefront sends out secondary wavelets to the screen. These wavelets then interfere to produce the diffraction pattern. It becomes simple to deal with rays instead of wavefronts as shown in the figure. In this figure, only nine rays have been drawn whereas actually there are a large number of them. Let us consider rays 1 and 5 which are in phase on the wavefront AB. When these reach the wavefront AC, ray 5 would have a path difference ab say equal to $\lambda/2$. Thus, when these two rays reach point P on the screen, they will interfere destructively. Similarly, all other pairs 2 and 6, 3

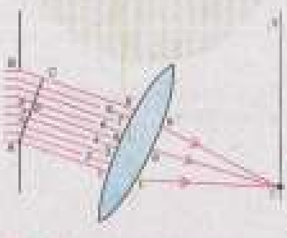


Fig. 9.9

Diffraction of light due to a narrow slit AB. The dots represent the sources of secondary wavelets.

and 7, 4 and 8 differ in path by $\lambda/2$ and will do the same. For the pairs of rays, the path difference ab = $d/2 \sin\theta$.

The equation for the first minimum is, then

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

or $$d\sin\theta = \lambda \qquad \text{..........} \qquad (9.7)$$

In general, the conditions for different orders of minima on either side of centre are given by

$$d\sin\theta = m\lambda \quad \text{where } m = \pm(1, 2, 3, \ldots) \qquad \text{..........} \qquad (9.8)$$

The region between any two consecutive minima both above and below O will be bright. A narrow slit, therefore, produces a series of bright and dark regions with the first bright region at the centre of the pattern. Such a diffraction pattern is shown in Fig. 9.10(a) and (b).

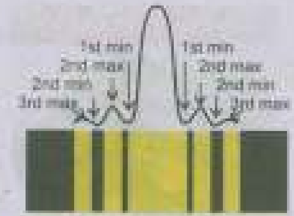


Fig. 9.10 (a)

Diffraction pattern of mono chromatic light produced due to a single slit, graphical representation and photograph of the pattern.


Fig. 9.10 (b)

Diffraction pattern produced by white light through a single slit.

9.10 DIFFRACTION GRATING

A diffraction grating is a glass plate having a large number of close parallel equidistant slits mechanically ruled on it. The transparent spacing between the scratches on the glass plate act as slits. A typical diffraction grating has about 400 to 5000 lines per centimetre.

In order to understand how a grating diffracts light, consider a parallel beam of monochromatic light illuminating the grating at normal incidence (Fig. 9.11). A few of the equally spaced narrow slits are shown in the figure. The distance between two adjacent slits is d, called grating element. Its value is obtained by dividing the length L of the grating by the total number N of the lines ruled on it. The sections of wave-front that pass through the slits behave as sources of secondary wavelets according to Huygen's principle.

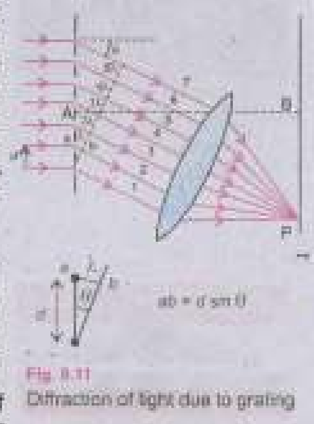


Fig. 9.11

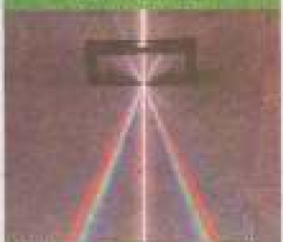
Diffraction of light due to grating

In Fig. 9.11, consider the parallel rays which after diffraction through the grating make an angle θ with AB, the normal to grating. They are then brought to focus on the screen at P by a convex lens. If the path difference between rays 1 and 2 is one wavelength λ, they will reinforce each other at P. As the incident beam consists of parallel rays, the rays from any two consecutive slits will differ in path by λ when they arrive at P. They will, therefore, interfere

Interesting Information

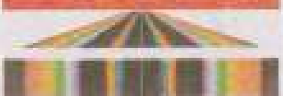
The fine rulings, each 0.5 μ m wide, on a compact disc function as a diffraction grating. When a small source of white light illuminates a disc, the diffracted light forms colored "lanes" that are composite of the diffraction patterns from the rulings.

Can You Tell?



Light waves projected through the diffraction grating produce an interference pattern. What colours are between the bands of interference?

For your Information



Diffraction of white light by a fine diffraction grating

constructively. Hence, the condition for constructive interference is that ab, the path difference between two consecutive rays, should be equal to λ i.e.,

$$ab = \lambda \qquad \text{..........} \qquad (9.9)$$

From Fig. 9.11

$$ab = d \sin\theta \qquad \text{..........} \qquad (9.10)$$

d being the grating element. Substituting the value of ab in Eq. 9.9

$$d \sin\theta = \lambda \qquad \text{..........} \qquad (9.11)$$

According to Eq. 9.10, when $\theta = 0$ i.e., along the direction of normal to the grating, the path difference between the rays coming out from the slits of the grating will be zero. So we will get a bright image in this direction. This is known as zero order image formed by the grating. If we increase θ on either side of this direction, a value of θ will be arrived at which $d\sin\theta$ will be equal to λ and according to Eq. 9.11, we will again get a bright image. This is known as first order image of the grating. In this way if we continue increasing θ, we will get the second, third, etc. images on either side of the zero order image with dark regions in between. The second, third order bright images would occur according as $d \sin\theta$ becoming equal to 2λ, 3λ, etc. Thus Eq. 9.11 can be written in more general form as

$$d \sin\theta = n\lambda \qquad \text{..........} \qquad (9.12)$$

where $n = 0 \pm 1 \pm 2 \pm 3$ etc.

However, if the incident light contains different wavelengths, the image of each wavelength for a certain value of n is diffracted in a different direction. Thus, separate images are obtained corresponding to each wavelength or colour. Eq. 9.12 shows that the value of θ depends upon n, so the images of different colours are much separated in higher orders.

9.11 DIFFRACTION OF X-RAYS BY CRYSTALS

X-rays is a type of electromagnetic radiation of much shorter wavelength, typically of the order of 10^{-10} m.

In order to observe the effects of diffraction, the grating spacing must be of the order of the wavelength of the radiation used. The regular array of atoms in a crystal forms a natural diffraction grating with spacing that is typically $\approx 10^{-10}$ m. The scattering of X-rays from the atoms in a crystalline lattice gives rise to diffraction effects very similar to those observed with visible light incident on ordinary grating.

The study of atomic structure of crystals by X-rays was initiated in 1914 by W.H. Bragg and W.L. Bragg with remarkable achievements. They found that a monochromatic beam of X-rays was reflected from a crystal plane as if it acted like mirror. To understand this effect, a series of atomic planes of constant interplanar spacing d parallel to a crystal face are shown by lines PP', $P_1P'_1$, $P_2P'_2$ and so on, in Fig. 9.12.

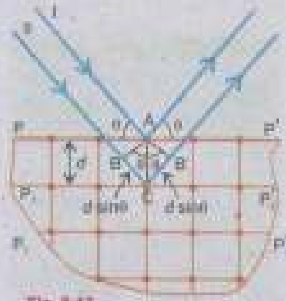


Fig. 9.12

Diffraction of X-rays from the lattice planes of crystal.

Suppose an X-rays beam is incident at an angle θ on one of the planes. The beam can be reflected from both the upper and the lower planes of atoms. The beam reflected from lower plane travels some extra distance as compared to the beam reflected from the upper plane. The effective path difference between the two reflected beams is $2d \sin\theta$. Therefore, for reinforcement, the path difference should be an integral multiple of the wavelength. Thus

$$2d \sin\theta = n\lambda \quad \text{..........} \quad (9.13)$$

The value of 'n' is referred to as the order of reflection. The equation 9.13 is known as the Bragg equation. It can be used to determine interplanar spacing between similar parallel planes of a crystal if X-rays of known wavelength are allowed to diffract from the crystal.

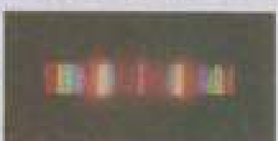
X-ray diffraction has been very useful in determining the structure of biologically important molecules such as haemoglobin, which is an important constituent of blood, and double helix structure of DNA.

Interesting Application

Diffraction of radio waves

Interesting Information

The spectrum of white light due to diffraction grating of 100 slits.



The spectrum of white light due to diffraction grating of 2000 slits.

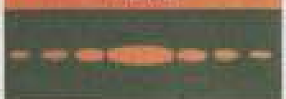
Example 9.3: Light of wavelength 450 nm is incident on a diffraction grating on which 5000 lines/cm have been ruled.

(i) How many orders of spectra can be observed on either side of the direct beam?

Interesting Illustration

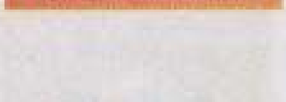
A multi-aperture diffraction pattern. This is a picture of a white-light point source shot through a piece of tightly woven cloth.

Tidbits



Diffraction pattern of a single human hair under laser beam illumination.

For Your Information



Looking through two polarizers. When they are "crossed", very little light passes through.

(ii) Determine the angle corresponding to each order.

Solution: (i) Given that

$$\lambda = 450\ \text{nm} = 450 \times 10^{-9}\ \text{m}$$

$$d = \frac{1}{5000}\text{cm} = \frac{1}{500000}\text{m}$$

For maximum number of order of spectra $\sin\theta = 1$

Since $\quad d\sin\theta = n\lambda$

therefore, subsituting the values in the above equation, we get

$$\frac{1}{500000}\text{m} \times 1 = n \times 450 \times 10^{-9}\text{m} \quad \text{or} \quad n = \frac{1}{500000 \times 450 \times 10^{-9}}$$

or $\quad n = 4.4$

Hence, the maximum order of spectrum is 4.

(i) For the first order of spectrum, n = 1.

$d\sin\theta = n\lambda$, gives

$$\frac{1}{500000}\text{m} \times \sin\theta = 1 \times 450 \times 10^{-9}\text{m}$$

$$\sin\theta = (500000)(450 \times 10^{-9})$$

$$\sin\theta = 0.225 \quad \text{or} \quad \theta = 13^{\circ}$$

For second order spectrum, n = 2, using Eq. $d\sin\theta = n\lambda$

$$\left(\frac{1}{500000}\text{m}\right)\sin\theta = 2 \times (450 \times 10^{-9}\text{m})$$

$$\sin\theta = 0.45$$

or $\quad \theta = 26.7^{\circ}$

The third order spectrum (n=3) will be observed at $\theta = 42.5^{\circ}$

$$\sin\theta = 3 \times 500000\ \text{m}^{-1} \times 450 \times 10^{-9}\text{m}$$

$$= 0.675 \quad \text{i.e.} \quad \text{at } \theta = 42.5^{\circ}$$

and the fourth order spectrum (n = 4) will occur at $\theta = 64.2^{\circ}$

$$\sin\theta = 4 \times 500000\ \text{m}^{-1} \times 450 \times 10^{-9}\text{m}$$

$$\sin\theta = 0.9 \quad \text{gives} \quad \theta = 64.2^{\circ}$$

9.12 POLARIZATION

In transverse mechanical waves, such as produced in a stretched string, the vibrations of the particles of the medium are perpendicular to the direction of propagation of the waves. The vibration can be oriented along vertical, horizontal or any other direction (Fig. 9.13). In each of these cases, the transverse mechanical wave is said to be polarized. The plane of polarization is the plane containing the direction of vibration of the particles of the medium and the direction of propagation of the wave.

A light wave produced by oscillating charge consists of a periodic variation of electric field vector accompanied by thermagnetic field vector at right angle to each other. Ordinary light has components of vibration in all possible planes. Such a light is unpolarized. On the other hand, if the vibrations are confined only in one plane, the light is said to be polarized.

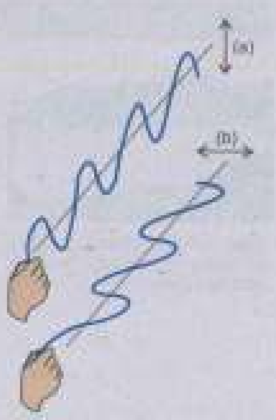


Fig. 9.13
Transverse waves on a string polarized (a) in a vertical plane and (b) in a horizontal plane

Production and Detection of Plane Polarized Light

The light emitted by an ordinary incandescent bulb (and also by the Sun) is unpolarized, because its (electrical) vibrations are randomly oriented in space (Fig. 9.14). It is possible to obtain plane polarized beam of light from un-polarized light by removing all waves from the beam except those having vibrations along one particular direction. This can be achieved by various processes such as selective absorption, reflection from different surfaces, refraction through crystals and scattering by small particles.

Fig. 9.14
An unpolarized light, due to incandescent bulb, has vibrations in all directions.

The selective absorption method is the most common method to obtain plane polarized light by using certain types of materials called dichroic substances. These materials transmit only those waves, whose vibrations are parallel to a particular direction anb will absorb those waves whose vibrations are in other directions. One such commercial polarizing material is a polaroid.

If un-polarized light is made incident on a sheet of polaroid, the transmitted light will be plane polarized. If a second sheet of polaroid is•placed in such a way that the axes of the polaroids, shown by straight lines drawn on them, are parallel (Fig. 9.15a), the light is transmitted through the second polaroid also. If the second polaroid is slowly rotated about the beam of light, as axis of rotation, the light emerging out of the second polaroid gets dimmer and dimmer and disappears when the axes become mutually

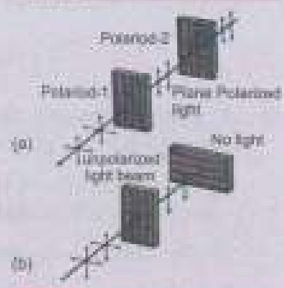
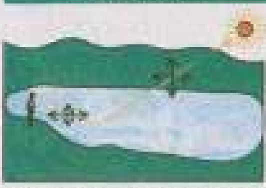


Fig. 9.15
Experimental arrangement to show that light waves are transverse. The lines with arrows indicates electric vibrations of light waves.

Do you know?



Light reflected from smooth surface of water is partially polarized parallel to the surface.

perpendicular (Fig. 9.15 b). The light reappears on further rotation and becomes brightest when the axes are again parallel to each other.

This experiment proves that light waves are transverse waves. If the light waves were longitudinal, they would never disappear even if the two polaroids were mutually perpendicular.

Reflection of light from water, glass, snow and rough road surfaces, for larger angles of incidences, produces glare. Since the reflected light is partially polarized, glare can considerably be reduced by using polaroid sunglasses.

Sunlight also becomes partially polorized because of scattering by air molecules of the Earth's atmosphere. This effect can be observed by looking directly up through a pair of sunglasses made of polarizing glass. At certain orientations of the lenses, less light passes through than at others.

Interesting Information

Sugar solution rotates the plane of polarization of incident light so that it is no longer horizontal but at an angle. The analyzer thus stops the light when rotated from the vertical (crossed) positions.

Optical Rotation

When a plane polarized light is passed through certain crystals, they rotate the plane of polarization. Quartz and sodium chlorate crystals are typical examples, which are termed as optically active crystals.

A few millimeter thickness of such crystals will rotate the plane of polarization by many degrees. Certain organic substances, such as sugar and tartaric acid, show optical rotation when they are in solution. This property of optically active substances can be used to determine their concentration in the solutions.

SUMMARY

- A surface passing through all the points undergoing a similar disturbance (i.e., having the same phase) at a given instant is called a wavefront.
- When the disturbance is propagated out in all directions from a point source, the wavefronts in this case are spherical.
- Radial lines leaving the point source in all directions represent rays.
- The distance between two consecutive wavefronts is called wavelength.
- Huygen's principle states that all points on a primary wavefront can be considered as the source of secondary wavelets.

- When two or more waves overlap each other, there is a resultant wave. This phenomenon is called interference.
- Constructive interference occurs when two waves, travelling in the same medium overlap and the amplitude of the resultant wave is greater than either of the individual waves.
- In case of destructive interference, the amplitude of the resulting wave is less than either of the individual waves.
- In Young's double slit experiment,

  (i) for bright fringe, $d\sin\theta = m\lambda$

  (ii) for dark fringe, $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$

  (iii) the distance between two adjacent bright or dark fringes is

$$\Delta y = \frac{L\lambda}{d}$$

- Newton's rings are circular fringes formed due to interference in a thin air film enclosed between a convex lens and a flat glass plate.
- Michelson's interferometer is used for very precise length measurements. The distance L of the moving mirror when m fringes move in view is $m\lambda/2$.
- Bending of light around obstacles is due to diffraction of light.
- For a diffraction grating:

  $d\sin\theta = n\lambda$ where n stands for nth order of maxima.
- For diffraction of X-rays by crystals

  $2d\sin\theta = n\lambda$ where n is the order of reflection.
- Polarization of light proves that light consists of transverse electromagnetic waves.

QUESTIONS

9.1 Under what conditions two or more sources of light behave as coherent sources?

9.2 How is the distance between interference fringes affected by the separation between the slits of Young's experiment? Can fringes disappear?

9.3 Can visible light produce interference fringes? Explain.

9.4 In the Young's experiment, one of the slits is covered with blue filter and other with red filter. What would be the pattern of light intensity on the screen?

9.5 Explain whether the Young's experiment is an experiment for studying interference or diffraction effects of light.

9.6 An oil film spreading over a wet footpath shows colours. Explain how does it happen?

9.7 Could you obtain Newton's rings with transmitted light? If yes, would the pattern be different from that obtained with reflected light?

9.8 In the white light spectrum obtained with a diffraction grating, the third order image of a wavelength coincides with the fourth order image of a second wavelength. Calculate the ratio of the two wavelengths.

9.9 How would you manage to get more orders of spectra using a diffraction grating?

9.10 Why the polaroid sunglasses are better than ordinary sunglasses?

9.11 How would you distinguish between un-polarized and plane-polarized lights?

9.12 Fill in the blanks.

(i) According to ________ principle, each point on a wavefront acts as a source of secondary ________.

(ii) In Young's experiment, the distance between two adjacent bright fringes for violet light is ________ than that for green light.

(iii) The distance between bright fringes in the interference pattern ________ as the wavelength of light used increases.

(iv) A diffraction grating is used to make a diffraction pattern for yellow light and then for red light. The distances between the red spots will be ________ than that for yellow light.

(v) The phenomenon of polarization of light reveals that light waves are ________ waves.

(vi) A polaroid is a commercial ________.

(vii) A polaroid glass ________ glare of light produced at a road surface.

NUMERICAL PROBLEMS

9.1 Light of wavelength 546 nm is allowed to illuminate the slits of Young's experiment. The separation between the slits is 0.10 mm and the distance of the screen from the slits where interference effects are observed is 20 cm. At what angle the first minimum will fall? What will be the linear distance on the screen between adjacent maxima?

(Ans: 0.16°, 1.1 mm)

9.2 Calculate the wavelength of light, which illuminates two slits 0.5 mm apart and produces an interference pattern on a screen placed 200 cm away from the slits. The first bright fringe is observed at a distance of 2.40 mm from the central bright image.

(Ans: 600 nm)

9.3 In a double slit experiment the second order maximum occurs at $\theta = 0.25°$. The wavelength is 650 nm. Determine the slit separation.

(Ans: 0.30 mm)

9.4 A monochromatic light of $\lambda = 588$ nm is allowed to fall on the half silvered glass plate G_1, in the Michelson Interferometer. If mirror M_1 is moved through 0.233 mm, how many fringes will be observed to shift?

(Ans: 792)

9.5 A second order spectrum is formed at an angle of 38.0° when light falls normally on a diffraction grating having 5400 lines per centimetre. Determine wavelength of the light used.

(Ans. 570 nm)

9.6 A light is incident normally on a grating which has 2500 lines per centimetre. Compute the wavelength of a spectral line for which the deviation in second order is 15.0°.

(Ans: 518 nm)

9.7 Sodium light ($\lambda = 589$ nm) is incident normally on a grating having 3000 lines per centimetre. What is the highest order of the spectrum obtained with this grating?

(Ans: 5th)

9.8 Blue light of wavelength 480 nm illuminates a diffraction grating. The second order image is formed at an angle of 30° from the central image. How many lines in a centimetre of the grating have been ruled?

(Ans: 5.2×10^3 lines per cm)

9.9 X-rays of wavelength 0.150 nm are observed to undergo a first order reflection at a Bragg angle of 13.3° from a quartz (SiO_2) crystal. What is the interplanar spacing of the reflecting planes in the crystal?

(Ans: 0.326 nm)

9.10 An X-ray beam of wavelength λ undergoes a first order reflection from a crystal when its angle of incidence to a crystal face is 26.5°, and an X-ray beam of wavelength 0.097 nm undergoes a third order reflection when its angle of incidence to that face is 60.0°. Assuming that the two beams reflect from the same family of planes, calculate (a) the interplanar spacing of the planes and (b) the wavelength λ.

[Ans: (a) 0.168 nm (b) 0.150 nm]

Chapter 10

OPTICAL INSTRUMENTS

Learning Objectives

At the end of this chapter the students will be able to:

1. Recognize the term of least distance of distinct vision.
2. Understand the terms magnifying power and resolving power.
3. Derive expressions for magnifying power of simple microscope, compound microscope and astronomical telescope.
4. Understand the working of spectrometer.
5. Describe Michelson rotating mirror method to find the speed of light.
6. Understand the principles of optical fibre.
7. Identify the types of optical fibres.
8. Appreciate the applications of optical fibres.

In this chapter, some optical instruments that are based on the principles of reflection and refraction, will be discussed. The most common of these instruments are the magnifying glass, compound microscope and telescopes. We shall also study magnification and resolving powers of these optical instruments. The spectrometer and an arrangement for measurement of speed of light are also described. An introduction to optical fibres, which has developed a great importance in medical diagnostics, telecommunication and computer networking, is also included.

10.1 LEAST DISTANCE OF DISTINCT VISION

The normal human eye can focus a sharp image of an object on the eye if the object is located any where from infinity to a certain point called the near point.

The minimum distance from the eye at which an object appears to be distinct is called the least distance of distinct vision or near point.

This distance represented by d is about 25 cm from the eye. If the object is held closer to the eye than this distance the image formed will be blurred and fuzzy. The location of the near point, however, changes with age.

10.2 MAGNIFYING POWER AND RESOLVING POWER OF OPTICAL INSTRUMENTS

When an object is placed in front of a convex lens at a point beyond its focus, a real and inverted image of the object is formed as shown in the Fig. 10.1.

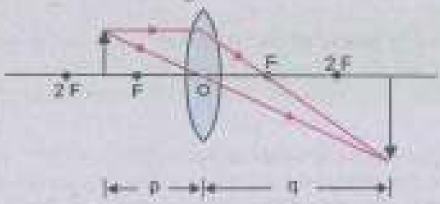


Fig. 10.1

The ratio of the size of the image to the size of the object is called magnification.

As the object is brought from a far off point to the focus, the magnification goes on increasing. The apparent size of an object depends on the angle subtended by it at the eye. Thus, the closer the object is to the eye, the greater is the angle subtended and larger appears the size of the object (Fig.10.2). The maximum size of an object as seen by naked eye is obtained when the object is placed at the least distance of distinct vision. For lesser distance, the image formed looks blurred and the details of the object are not visible.

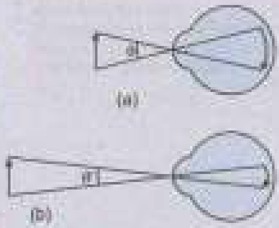


Fig. 10.2

When the same object is viewed at a shorter distance, the image on the retina of the eye is greater; so the object appears larger and more details can be seen. The angle α the object subtends in (a) is greater than θ in (b).

The magnifying power or angular magnification is defined as the ratio of the angles subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.

The optical resolution of a microscope or a telescope tells us how close together the two point sources of light can be so that they are still seen as two separate sources. If two point sources are too close, they will appear as one because the optical instrument makes a point source look like a small disc or spot of light with circular diffraction fringes.

Although the magnification can be made as large as one desires by choosing appropriate focal lengths, but the magnification alone is of no use unless we can see the details of the object distinctly.

The resolving power of an instrument is its ability to reveal the minor details of the object under examination.

Tid-bits

If you find it difficult to read small print, make a pinhole in a piece of paper and hold it in front of your eye close to the page. You will see the print clearly.

Resolving power is expressed as the reciprocal of minimum angle which two point sources subtends at the instrument so that their images are seen as two distinct spots of light rather than one. Raleigh showed that for light of wavelength λ through a lens of diameter D, the resolving power is given by $R = \frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda}$

Where

$$\alpha_{min} = 1.22\frac{\lambda}{D} \qquad \cdots\cdots\cdots (10.1)$$

The smaller the value of α_{min}, greater is the resolving power because two distant objects which are close together can then be seen separated through the instrument. In the case of a grating spectrometer, the resolving power R of the grating is defined as

$$R = \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta\lambda} \qquad \cdots\cdots\cdots (10.2)$$

where $\lambda \approx \lambda_1 \approx \lambda_2$ and $\Delta\lambda = \lambda_2 - \lambda_1$. Thus, we see that a grating with high resolving power can distinguish small difference in wavelength. If N is the number of rulings on the grating, it can be shown that the resolving power in the mth-order diffraction equals the product N x m, i.e.

$$R = N \times m \qquad \cdots\cdots\cdots (10.3)$$

10.3 SIMPLE MICROSCOPE

As discussed above, a converging or convex lens can be used to help the eye to see small objects distinctly. A watch maker uses convex lens to repair the watches. The object is placed inside the focal point of the lens. The magnified and virtual image is formed at least distance of distinct vision *d* or much farther from the lens.

Let us, now, calculate the magnification of a simple microscope. In Fig. 10.3 (a), the image formed by the object, when placed at a distance *d*, on the eye is shown. In Fig. 10.3 (b), a lens is placed just in front of the eye and the object is placed in front of the lens in such a way that a virtual image of the object is formed at a distance *d* from the eye. The size of the image is now much larger than without the lens.

If β and α are the respective angles subtended by the object when seen through the lens (simple microscope) and when viewed directly, then angular magnification *M* is defined as

$$M = \frac{\beta}{\alpha} \qquad \cdots\cdots \qquad (10.4)$$

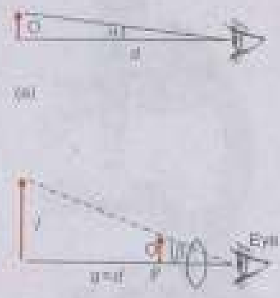


Fig. 10.3
Simple Microscope

When angles are small, then they are nearly equal to their tangents. From Fig. 10.3 (a) and (b), we find

$$\alpha = \tan\alpha = \frac{\text{Size of the object}}{\text{Distance of the object}} = \frac{O}{d}$$

and $$\beta = \tan\beta = \frac{\text{Size of the image}}{\text{Distance of the image}} = \frac{I}{q}$$

Since the image is at the least distance of distinct vision,

hence, $q = d$

Therefore, $\beta = \frac{I}{q} = \frac{I}{d}$

the angular magnification $M = \frac{I/d}{O/d} = \frac{I}{O}$

As we already know that

$$\frac{I}{O} = \frac{\text{Size of the image}}{\text{Size of the object}} = \frac{\text{Distance of the image}}{\text{Distance of the object}} = \frac{q}{p}$$

Therefore, $$M = \frac{q}{p} = \frac{d}{p} \quad \text{............ (10.5)}$$

For virtual image, the lens formula is written as

$$\frac{1}{f} = \frac{1}{p} - \frac{1}{q} \qquad \text{But} \qquad q = d$$

Hence $$\frac{1}{f} = \frac{1}{p} - \frac{1}{d} \qquad \text{or} \qquad \frac{d}{p} = 1 + \frac{d}{f}$$

Hence the magnification of a convex lens (simple microscope) can be expressed as

$$M = \frac{d}{p} = 1 + \frac{d}{f} \quad \text{............ (10.6)}$$

It is, thus, obvious that for a lens of high angular magnification the focal length should be small. If, for example, f = 5 cm and d = 25 cm, then M = 6, the object would look six times larger when viewed through such a lens.

Fig.10.4 (b).
A Compound Microscope

10.4 COMPOUND MICROSCOPE

Whenever high magnification is desired, a compound microscope is used. It consists of two convex lenses, an object lens of very short focal length and an eye-piece of comparatively longer focal length. The ray diagram of a compound microscope is given in Fig. 10.4 (a).

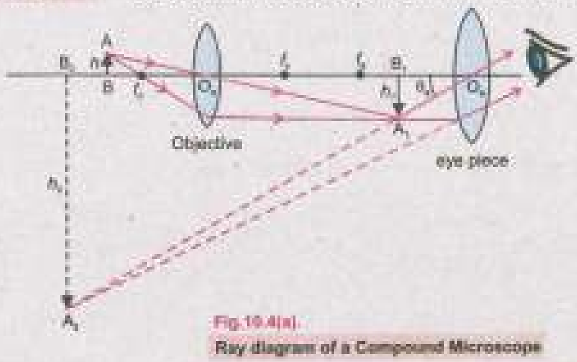


Fig.10.4(a).
Ray diagram of a Compound Microscope

The object of height h is placed just beyond the principal focus of the objective. This produces a real, magnified image of height h_1 of the object at a place situated within the focal point of the eye-piece. It is then further magnified by the eye-piece. In normal adjustment, the eye-piece is positioned so that the final image is formed at the near point of the eye at a distance d.

The angular magnification M of a compound microscope is defined to be the ratio $\tan\theta_e/\tan\theta$, where θ_e is the angle subtended by the final image of height h_2 and θ is the angle that the object of height h would subtend at the eye if placed at the near point d (Fig. 10.3 a). Now

$$\tan\theta = \frac{h}{d} \qquad \text{and} \qquad \tan\theta_e = \frac{h_2}{d}$$

Thus, magnification $$M = \frac{\tan\theta_e}{\tan\theta} = \frac{h_2}{d} \times \frac{d}{h} = \frac{h_2}{h}$$

or $$M = \frac{h_1}{h} \times \frac{h_2}{h_1}$$

where ratio h_1/h is the linear magnification M_1 of the objective and h_2/h_1 is the magnification M_2 of the eyepiece. Hence, total magnification is

$$M = M_1 M_2$$

By Eq. 10.5 and Eq. 10.6, $M_1 = q/p$ and $M_2 = 1 + d/f_e$

Hence, $$M = \frac{q}{p}\left(1 + \frac{d}{f_e}\right) \cdots\cdots\cdots (10.7)$$

It is customary to refer the values of M as multiples of 5, 10, 40 etc., and are marked as ×5, ×10, ×40 etc., on the instrument.

The limit to which a microscope can be used to resolve details, depends on the width of the objective. A wider objective and use of blue light of short wavelength produces less diffraction and allows more details to be seen.

For Your Information

A seventeenth century microscope which could be moved up and down in its support ring (Courtesy of the Museum of the History of Science, Florence).

Example 10.1: A microscope has an objective lens of 10 mm focal length, and an eye piece of 25.0 mm focal length. What is the distance between the lenses and its magnification, if the object is in sharp focus when it is 10.5 mm from the objective?

Solution: If we consider the objective alone

$$\frac{1}{10.5\,\text{mm}}+\frac{1}{q}=\frac{1}{10\,\text{mm}} \qquad \text{or} \qquad q = 210\ \text{mm}$$

If we consider the eye piece alone, with the virtual image at the least distance of distinct vision $d = -250$ mm

$$\frac{1}{p}+\frac{1}{-250\,\text{mm}}=\frac{1}{25\,\text{mm}} \qquad \text{or} \qquad p = 22.7\ \text{mm}$$

Distance between Lenses $= q + p = 210\text{mm} + 22.7\text{mm} = 233\text{mm}$

Magnification by objective

$$M_1=\frac{q}{p}=\frac{210\,\text{mm}}{10.5\,\text{mm}}=20.0$$

Magnification by eye piece

$$M_2=\frac{-250\,\text{mm}}{22.7\,\text{mm}}=-11.0$$

Total magnification

$$M = M_1 \times M_2$$
$$= 20 \times (-11.0) = -220$$

-ive sign indicates that the image is virtual.

10.5 ASTRONOMICAL TELESCOPE

Telescope is an optical device used for viewing distant objects. The image of a distant object viewed through a telescope appears larger because it subtends a bigger visual angle than when viewed with the naked eye. Initially the extensive use of the telescopes was for astronomical observations. These telescopes are called astronomical telescopes. A simple astronomical telescope consists of two convex lenses, an objective of long focal length f_o and an eye piece of short focal length f_e. The objective forms a real, inverted and diminished image $A'B'$ of a distant object AB. This real image $A'B'$ acts as object for the eye piece which is used as a magnifying glass. The final image seen through the eye-piece is virtual, enlarged and inverted. Fig. 10.5 shows the path of rays through an astronomical telescope.

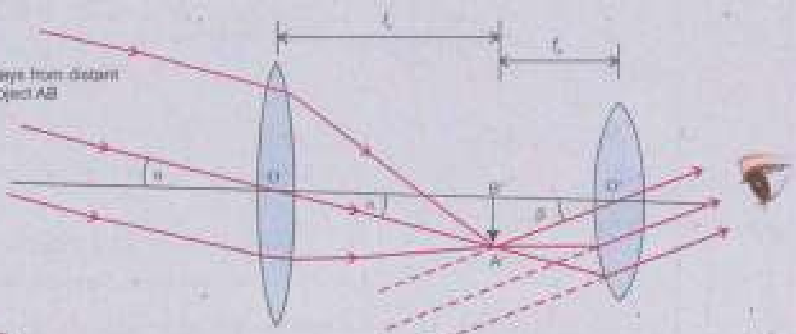


Fig 10.5
Ray diagram of Astronomical Telescope

When a very distant object is viewed, the rays of light coming from any of its point (say its top) are considered parallel and these parallel rays are converged by the objective to form a real image $A'B'$ at its focus. If it is desired to see the final image through the eye-piece without any strain on the eye, the eye-piece must be placed so that the image $A'B'$ lies at its focus. The rays after refraction through the eye-piece will become parallel and the final image appears to be formed at infinity. In this condition the image $A'B'$ formed by the objective lies at the focus of both the objective and the eye-piece and the telescope is said to be in normal adjustment.

Let us now compute the magnifying power of an astronomical telescope in normal adjustment. The angle α subtended at the unaided eye is practically the same as subtended at the objective and it is equal to $\angle A'OB'$. Thus

$$\alpha = \tan\alpha = \frac{A'B'}{OB'} = \frac{A'B'}{f_o}$$

The angle β subtended at the eye by the final image is equal to ∠ A′ O′ B′ . Thus

$$\beta = \tan\beta = \frac{A'B'}{O'B'} = \frac{A'B'}{f_e}$$

Magnifying power of the telescope $= \frac{\beta}{\alpha} = \frac{A'B'/f_e}{A'B'/f_o}$

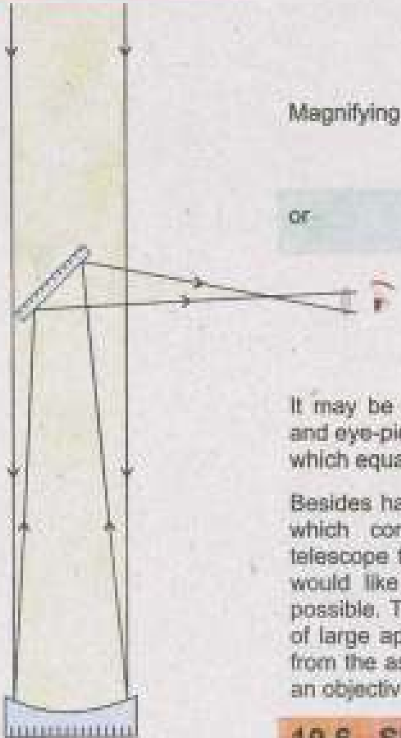
or $$M = \frac{f_o}{f_e} \qquad \text{..........} \qquad (10.8)$$

$$M = \frac{\text{Focal length of the objective}}{\text{Focal length of the eyepiece}}$$

It may be noted that the distance between the objective and eye-piece of a telescope in normal adjustment is $f_o + f_e$ which equals the length of the telescope.

Besides having a high magnifying power another problem which confronts the astronomers while designing a telescope to see the distant planets and stars is that they would like to gather as much light form the object as possible. This difficulty is overcome by using the objective of large aperture so that it collects a great amount of light from the astronomical objects. Thus a good telescope has an objective of long focal length and large aperture.

For Your Information



Reflecting Telescope

Large astronomical telescopes are reflecting type made from specially shaped very large mirrors used as objectives. With such telescopes, astronomers can study stars which are millions light year away.

10.6 SPECTROMETER

A spectrometer is an optical device used to study spectra from different sources of light. With the help of a spectrometer, the deviation of light by a glass prism and the refractive index of the material of the prism can be measured quite accurately. Using a diffraction grating, the spectrometer can be employed to measure the wave length of the light.

The essential components of a spectrometer are shown in Fig. 10.6 (a).

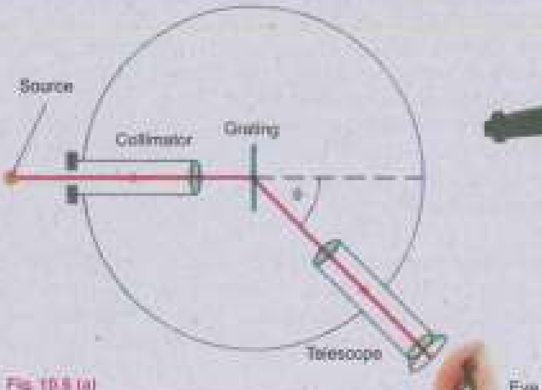


Fig 10.8 (a)

Schematic diagram of a spectrometer.

Fig 10.8 (b)

Spectrometer.

Collimator

It consists of a fixed metallic tube with a convex lens at one end and an adjustable slit, that can slide in and out of the tube, at the other end. When the slit is just at the focus of the convex lens, the rays of light coming out of the lens become parallel. For this reason, it is called a collimator.

Turn Table

A prism or a grating is placed on a turn table which is capable of rotating about a fixed vertical axis. A circular scale, graduated in half degrees, is attached with it.

Telescope

A telescope is attached with a vernier scale and is rotatable about the same vertical axis as the turn table.

Before using a spectrometer, one should be sure that the collimator is so adjusted that parallel rays of light emerge out of its convex lens. The telescope is adjusted in such a way that the rays of light entering it are focussed at the cross wires near the eye-piece. Finally, the refracting edge of the prism must be parallel to the axis of rotation of the telescope so that the turn table is levelled. This can be done by using the levelling screws.

10.7 SPEED OF LIGHT

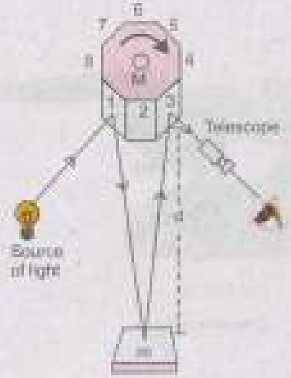


Fig. 10.7

Michelson's method for measurement of speed of light.

Light travels so rapidly that it is very difficult to measure its speed. Galileo was the first person to make an attempt to measure its speed. Although he did not succeed in the measurement of the speed of light, yet he was convinced that the light does take some time to travel from one place to another. Given below is one of the accurate methods of determining the speed of light which is known as Michelson's experiment.

In this experiment, the speed of light was determined by measuring the time it took to cover a round trip between two mountains. The distance between the two mountains was measured accurately. The experimental set up is shown in Fig 10.7.

An eight-sided polished mirror M is mounted on the shaft of a motor whose velocity can be varied. Suppose the mirror is stationary in the position shown in the figure. A beam of light from the face 1 of the mirror M falls at the plane mirror m placed at a distance *d* from M. The beam is reflected back from the mirror m and falls on the face 3 of the mirror M. On reflection from face 3, it enters the telescope.

If the mirror M is rotated clockwise, initially the source will not be visible through the telescope. When the mirror M gains a certain speed, the source *S* becomes visible. This happens when the time taken by light in moving from M to m and back to M is equal to the time taken by face 2 to move to the position of face 1.

Angle subtended by any side of the eight-sided mirror at the centre is $2\pi/8$. If f is the frequency of the mirror M, when the source *S* is visible through the telescope, then the time taken by the mirror to rotate through an angle 2π is $1/f$. So, the time taken by the mirror M to rotate through an angle $2\pi/8$ is

$$t = \frac{1}{2\pi f} \times \frac{2\pi}{8} = \frac{1}{8f}$$

The time taken by light for its passage from M to *m* and back is $2d/c$, where *c* is the speed of light. These two times are equal.

so
$$\frac{1}{8f} = \frac{2d}{c}$$

$$c = 16fd \quad \cdots\cdots\cdots \quad (10.9)$$

This equation was used to determine the speed of light by Michelson. Presently accepted value for the speed of light in vacuum is

$$c = 2.99792458 \times 10^8 \text{ ms}^{-1}$$

we usually round this off to $3.00 \times 10^8 \text{ ms}^{-1}$.

The speed of light in other materials is always less than c. In media other than vacuum, it depends upon the nature of the medium. However, the speed of light in air is approximately equal to that in vacuum and generally taken so in calculations.

10.8 INTRODUCTION TO FIBRE OPTICS

For hundreds of years man has communicated using flashes of reflected sunlight by day and lanterns by night. Navy signalmen still use powerful blinker lights to transmit coded messages to other ships during periods of radio-silence. Light communication has not been confined to simple dots and dashes. It is an interesting but little known fact that Alexander Graham Bell invented a device known as "photo phone" shortly after his invention of telephone. Bell's photo phone used a modulated beam of reflected sunlight, focussed upon a Selenium detector several hundred metres away. With the device, Bell was able to transmit a voice message via a beam of light. The idea remained dormant for many years. During the recent past the idea of transmission of light through thin optical fibres has been revived and is now being used in communication technology.

Point to Ponder

Each of the thin optical fibres is small enough to fit through the eye of a needle. Why is the size of the fibre important?

The use of light as a transmission carrier wave in fibre optics has several advantages over radio wave carriers such as a much wider bandwidth capability and immunity from electromagnetic interference.

Fig. 10.8 (a)

Optical fibre image

It is also used to transmit light around corners and into inaccessible places so that the formerly unobservable could be viewed. The use of fibre optic tools in industry is now very common, and their importance as diagnostic tools in medicine has been proved (Fig. 10.8 a and b).

Recently the fibre optic technology has evolved into something much more important and useful – a communication system of enormous capabilities.

One feature of such a system is its ability to transmit thousands of telephone conversations, several television programs and numerous data signals between stations through one or two flexible, hair - thin threads of optical fibre. With the tremendous information carrying capacity called the bandwidth, fibre optic systems have undoubtedly made practical such services as two way television which was too costly before the development of fibre optics. These systems also allow word processing, image transmitting and receiving equipment to operate efficiently.

Fig. 10.8 (b)

A precision diamond scalpel for use in eye surgery. The illumination is obtained by light passing through a fibre optic light guide

In addition to giving an extremely wide bandwidth, the fibre optic system has much thinner and light weight cables. An optical fibre with its protective case may be typically 6.0 mm in diameter, and yet it can replace a 7.62 cm diameter bundle of copper wires now used to carry the same amount of signals.

10.9 FIBRE OPTIC PRINCIPLES

Propagation of light in an optical fibre requires that the light should be totally confined within the fibre.

This may be done by total internal reflection and continuous refraction.

Total Internal Reflection

One of the qualities of any optically transparent material is the speed at which light travels within the material, i.e., it depends upon the refractive index n of the material. The index of refraction is merely the ratio of the speed of light c in vacuum to the speed of light v in that material.

Expressed mathematically,

$$n = \frac{c}{v} \qquad \text{..........} \qquad (10.10)$$

The boundary between two optical media, e.g. glass and air having different refractive indices can reflect or refract light rays. The amount and direction of reflection or refraction is determined by the amount of difference in refractive indices as well as the angle at which the rays strike the boundary. At some angle of incidence, the angle of refraction is equal to 90° when a ray of light is passing through glass to air. This angle of incidence is called the critical angle θ_c shown in Fig.10.9(a).We are already familiar with Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

From Fig. 10.9 (a), when $\theta_1 = \theta_c$, $\theta_2 = 90°$

thus, $n_1 \sin\theta_c = n_2$ or $\sin\theta_c = n_2 / n_1$

For incident angles equal to or greater than the critical angle, the glass - air boundary will act as a mirror and no light escapes from the glass (Fig. 10.9 b). For glass-air boundary, we have $\sin\theta_c = \frac{n_2}{n_1} = \frac{1.0}{1.5}$ or $\theta_c = 41.8°$

Fig. 10.9 (a)

If the angle of refraction in the air is 90° the angle of incidence is called the critical angle.

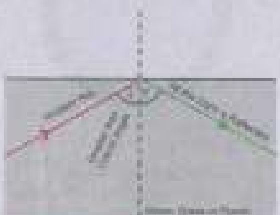


Fig.10.9(b)

For angles of incidence greater than the critical angle, all the light is reflected; none is refracted into the air.

Let us now assume that the glass is formed into a long, round rod. We know that all the light rays striking the internal surface of the glass at angles of incidence greater than 41.8° (critical angle) will be reflected back into the glass, while those with angles less than 41.8° will escape from the glass (Fig.10.10a). Ray 1 is injected into the rod so that it strikes the glass air boundary at an angle of incidence about 30°.

Fig. 10.10 (a)

Propagation of light within a glass rod.

Since this is less than the critical angle, it will escape from the rod and be lost. Ray 2 at 42° will be reflected back into the rod, as will ray 3 at 60°.Since the angle of reflection equals the angle of incidence, these two rays will continue to propagate down the rod, along paths determined by the original angles of incidence. Ray 4 is called an axial

Fig. 10.10 (b)

Light propagation within a flexible glass fibre

ray since its path is parallel to the axis of the rod. Axial rays will travel directly down this straight and rigid rod. However, in a flexible glass fibre they will be subjected to the laws of reflection (Fig.10.10b).

Optical fibres that propagate light by total internal reflection are the most widely used.

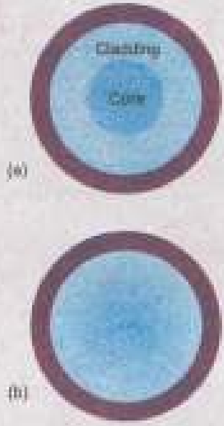


Fig. 10.11
Cross sectional view of
(a) Multi-mode step index fibre
(b) Multi-mode graded index fibre

Continuous Refraction

There is another mode of propagation of light through optical fibres in which light is continuously refracted within the fibre. For this purpose central core has high refractive index (high density) and over it is a layer of a lower refractive index (less density). This layer is called cladding. Such a type of fibre is called multi-mode step index fibre whose cross sectional view is shown in Fig.10.11(a).

Now a days, a new type of optical fibre is used in which the central core has high refractive index (high density) and its density gradually decreases towards its periphery. This type of optical fibre is called a multi mode graded index fibre. Its cross sectional view is shown in Fig. 10.11 (b).


Fig.10.12
Light propagation within a hypothetical multi layer fibre.

In both these fibres the propagation of light signal is through continuous refraction. We already know that a ray passing from a denser medium to a rarer medium bends away from the normal and vice versa. In step index or graded index fibre, a ray of light entering the optical fibre, as shown in Fig. 10.12, is continuously refracted through these steps and is reflected from the surface of the outer layer. Hence light is transmitted by continuous refraction and total internal reflection.

10.10 TYPES OF OPTICAL FIBRES

There are three types of optical fibres which are classified on the basis of the mode by which they propagate light. These are (i) single mode step index (ii) multi mode step index and (iii) multi mode graded index. The term 'mode' is described as the method by which light is propagated within the fibre, i.e. the various paths that light can take in travelling down the fibre. The optical fibre is also covered by a plastic jacket for protection.

(i) Single Mode Step Index Fibre

Single mode or mono mode step index fibre has a very thin core of about 5 μm diameter and has a relatively larger cladding (of glass or plastic) as shown in Fig. 10.13. Since it has a very thin core, a strong monochromatic light source i.e., a Laser source has to be used to send light signals through it. It can carry more than 14 TV channels or 14000 phone calls.

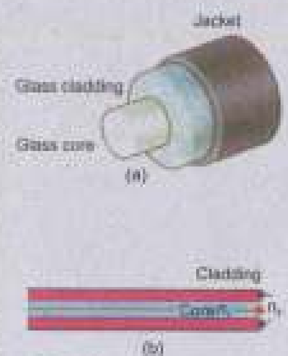


Fig. 10.13
Single-mode step-index fibre.

(ii) Multimode Step Index Fibre

This type of fibre has a core of relatively larger diameter such as 50 μm. It is mostly used for carrying white light but due to dispersion effects, it is useful for a short distance only. The fibre core has a constant refractive index n_1, such as 1.52, from its centre to the boundary with the cladding as shown in Fig. 10.14. The refractive index then changes to a lower value n_2, such as 1.48, which remains constant throughout the cladding.

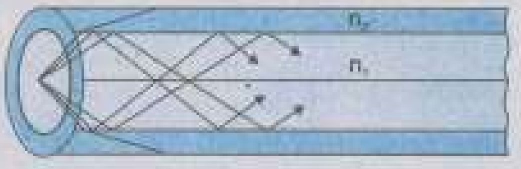


Fig. 10.14
Light propogation through Multi-mode step-index fibre.

This is called a step-index multimode fibre, because the refractive index steps down from 1.52 to 1.48 at the boundary with the cladding.

(iii) Multimode Graded Index Fibre

Multi mode graded index fibre has core which ranges in diameter from 50 to 1000 μm. It has a core of relatively high refractive index and the refractive index decreases gradually from the middle to the outer surface of the fibre. There is no noticeable boundary between core and cladding. This type of fibre is called a multi mode graded-index fibre (Fig. 10.15) and is useful for long distance applications in which white light is used. The mode of transmission of light through this type of fibre is also the same, i.e., continuous refraction from

Fig. 10.15
Light propogation through Multi-mode graded-index fibre

the surfaces of smoothly decreasing refractive index and the total internal reflection from the boundary of the outer surfaces.

Example 10.2: Calculate the critical angle and angle of entry for an optical fibre having core of refractive index 1.50 and cladding of refractive index 1.48.

Solution: We have $n_1 = 1.50$, $n_2 = 1.48$

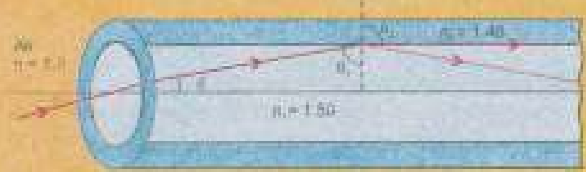


Fig. 10.16

From Snell's law $\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$

When $\qquad \theta_1 = \theta_c,\ \theta_2 = 90°$

So $\qquad 1.50 \sin\theta_c = 1.48 \sin 90°$

Which gives $\qquad \theta_c = 80.6°$

From the Fig. 10.16, $\qquad \theta' = 90° - \theta_c = 9.4°$

Again using Snell's law, we have $\qquad \dfrac{\sin\theta}{\sin\theta'} = \dfrac{n_1}{n} = \dfrac{1.5}{1}$

which gives $\qquad \sin\theta = 1.50 \sin\theta'$ or $\theta = 14.2°$

If light beam is incident at the end of the optical fibre at an angle greater than 14.2°, total internal reflection would not take place.

10.11 SIGNAL TRANSMISSION AND CONVERSION TO SOUND

A fibre optic communication system consists of three major components: (i) a transmitter that converts electrical signals to light signals, (ii) an optical fibre for guiding the signals and (iii) a receiver that captures the light signals at the other end of the fibre and reconverts them to electric signals.

The light source in the transmitter can be either a semiconductor laser or a light emitting diode (LED). With either device, the light emitted is an invisible infra-red signals. The typical wavelength is 1.3 µm.

Such a light will travel much faster through optical fibres than will either visible or ultra-violet light. The lasers and LEDs used in this application are tiny units (less than half the size of the thumbnail) in order to match the size of the fibres. To transmit information by light waves, whether it is an audio signal, a telvision signal or a computer data signal, it is necessary to modulate the light waves. The most common method of modulation is called digital modulation in which the laser or LED is flashed on and off at an extremely fast rate. A pulse of light represents the number 1 and the absence of light represents zero. In a sense, instead of flashes of light travelling down the fibre, ones (1s) and zeros (0s) are moving down the path.

Fig. 10.17

With computer type equipment, any communication can be represented by a particular pattern or code of these 1s and 0s. The receiver is programmed to decode the 1s and 0s, thus it receives, the sound, pictures or data as required. Digital modulation is expressed in bits (binary digit) or megabits (10^6 bits) per second, where a bit is a 1 or a 0.

Despite the ultra-purity (99.99% glass) of the optical fibre, the light signals eventually become dim and must be regenerated by devices called repeaters. Repeaters are typically placed about 30km apart, but in the newer systems they may be separated by as much as 100 km.

At the end of the fibre, a photodiode converts the light signals, which are then amplified and decoded, if necessary, to reconstruct the signals originally transmitted (Fig. 10.17).

10.12 LOSSES OF POWER

When a light signal travels along fibres by multiple reflection, some light is absorbed due to impurities in the glass. Some of it is scattered by groups of atoms which are formed at places such as joints when fibres are joined together. Careful manufacturing can reduce the power loss by scattering and absorption.

The information received at the other end of a fibre can be inaccurate due to dispersion or spreading of the light signal. Also the light signal may not be perfectly monochromatic. In such a case, a narrow band of wave-lengths are refracted in different directions when the light signal enters the glass fibre and the light spreads.

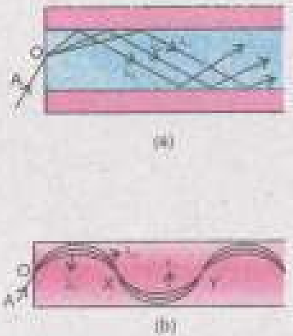


Fig. 10.18

Light paths in (a) step-index and (b) graded-index fibre.

Fig. 10.18 (a) shows the paths of light of three different wavelengths λ_1, λ_2 and λ_3. λ_1 meets the core-cladding at the critical angle and λ_2 and λ_3 at slightly greater angles. All the rays travel along the fibre by multiple reflections as explained earlier. But the light paths have different lengths. So the light of different wavelengths reaches the other end of the fibre at different times. The signal received is, therefore, faulty or distorted.

The disadvantage of the step-index fibre (Fig. 10.18 a) can considerably be reduced by using a graded index fibre. As shown in Fig. 10.18 (b), the different wavelengths still take different paths and are totally internally reflected at different layers, but still they are focussed at the same point like X and Y etc. It is possible because the speed is inversely-proportional to the refractive index. So the wavelength λ_1 travels a longer path than λ_2 or λ_3 but at a greater speed.

Inspite of the different dispersion, all the wavelengths arrive at the other end of the fibre at the same time. With a step-index fibre, the overall time difference may be about 33ns per km length of fibre. Using a graded index fibre, the time difference is reduced to about 1 ns per km.

SUMMARY

- Least distance of distinct vision is the minimum distance from the eye at which an object appears to be distinct.
- Magnification is the ratio of the size of the image to the size of the object, which equals to the ratio of the distance of the image to the distance of the object from the lens or mirror.
- Magnifying power or angular magnification is the angle subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.
- Resolving power is the ability of an instrument to reveal the minor details of the object under examination.

- Simple microscope is in fact a convex lens used to help the eye to see small objects distinctly. The magnifying power of a simple microscope is given by

$$M = \frac{d}{p} = 1 + \frac{d}{f}$$

- Compound microscope consists of two convex lenses, an objective lens of very short focal length and an eye piece of moderate focal length. The magnifying power of a compound microscope is given by

$$M = \frac{q}{p}\left(1 + \frac{d}{f_e}\right)$$

- Telescope is an optical instrument used to see distant object. The magnifying power of the telescope is given by

$$M = \frac{f_o}{f_e}$$

- Spectrometer is an optical device used to study spectra from different sources of light.
- Index of refraction is the ratio of speed of light in vacuum to the speed of light in the material.
- Critical angle is the angle of incidence in the denser medium for which the angle of refraction in the rarer medium is equal to 90°.
- When the angle of incidence becomes greater than the critical angle of that material, the incident ray is reflected in the same material, which is called total internal reflection.
- Cladding is a layer of lower refractive index (less density) over the central core of high refractive index (high density).
- Multi mode step index fibre is an optical fibre in which a layer of lower refractive index is over the central core of high refractive index.
- Multi mode graded index fibre is an optical fibre in which the central core has high refractive index and its density gradually decreases towards its periphery.

QUESTIONS

10.1 What do you understand by linear magnification and angular magnification? Explain how a convex lens is used as a magnifier?

10.2 Explain the difference between angular magnification and resolving power of an optical instrument. What limits the magnification of an optical instrument?

10.3 Why would it be advantageous to use blue light with a compound microscope?

10.4 One can buy a cheap microscope for use by the children. The images seen in such a microscope have coloured edges. Why is this so?

10.5 Describe with the help of diagrams, how (a) a single biconvex lens can be used as a magnifying glass. (b) biconvex lenses can be arranged to form a microscope.

10.6 If a person was looking through a telescope at the full moon, how would the appearance of the moon be changed by covering half of the objective lens.

10.7 A magnifying glass gives a five times enlarged image at a distance of 25 cm from the lens. Find, by ray diagram, the focal length of the lens.

10.8 Identify the correct answer.

(i) The resolving power of a compound microscope depends on:

a. Length of the microscope.

b. The diameter of the objective lens.

c. The diameter of the eyepiece.

d. The position of an observer's eye with regard to the eye lens.

(ii) The resolving power of an astronomical telescope depends on:

a. The focal length of the objective lens.

b. The least distance of distinct vision of the observer.

c. The focal length of the eye lens.

d. The diameter of the objective lens.

10.9 Draw sketches showing the different light paths through a single-mode and a multi mode fibre. Why is the single-mode fibre preferred in telecommunications?

10.10 How the light signal is transmitted through the optical fibre?

10.11 How the power is lost in optical fibre through dispersion? Explain.

NUMERICAL PROBLEMS

10.1 A converging lens of focal length 5.0 cm is used as a magnifying glass. If the near point of the observer is 25 cm and the lens is held close to the eye, calculate (i) the distance of the object from the lens (ii) the angular magnification. What is the angular magnification when the final image is formed at infinity?

[Ans: (i) 4.2 cm (ii) 6.0 ; 5.0]

10.2 A telescope objective has focal length 96 cm and diameter 12 cm. Calculate the focal length and minimum diameter of a simple eye piece lens for use with the telescope, if the linear magnification required is 24 times and all the light transmitted by the objective from a distant point on the telescope axis is to fall on the eye piece.

(Ans: f_e = 4.0 cm, dia = 0.50 cm)

10.3 A telescope is made of an objective of focal length 20 cm and an eye piece of 5.0cm, both convex lenses. Find the angular magnification.

(Ans: 4.0)

10.4 A simple astronomical telescope in normal adjustment has an objective of focal length 100 cm and an eye piece of focal length 5.0 cm. (i) Where is the final image formed ? (ii) Calculate the angular magnification.

[Ans: (i) infinity (ii) 20]

10.5 A point object is placed on the axis of and 3.6 cm from a thin convex lens of focal length 3.0 cm . A second thin convex lens of focal length 16.0 cm is placed coaxial with the first and 26.0 cm from it on the side away from the object. Find the position of the final image produced by the two lenses.

(Ans: 16 cm from second lens)

10.6 A compound microscope has lenses of focal length 1.0 cm and 3.0 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed, 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument.

(Ans: 8.7 cm, 47)

10.7 Sodium light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective is 0.90 cm, (i) find the limiting angle of resolution, (ii) using visible light of any wavelength, what is the maximum limit of resolution for this microscope.

[Ans: (i) 8.0×10^{-5} rad; (ii) 5.4×10^{-5} rad]

10.8 An astronomical telescope having magnifying power of 5 consist of two thin lenses 24 cm apart. Find the focal lengths of the lenses.

[Ans: 20 cm, 4 cm]

10.9 A glass light pipe in air will totally internally reflect a light ray if its angle of incidence is at least 39°. What is the minimum angle for total internal reflection if pipe is in water? (Refractive Index of water = 1.33)

[Ans: 57°]

10.10 The refractive index of the core and cladding of an optical fibre are 1.6 and 1.4 respectively. Calculate (i) the critical angle for the interface (ii) the maximum angle of incidence in the air of a ray which enters the fibre and is incident at the critical angle on the interface.

[Ans: (i) 61°, (ii) 51°]

Chapter 11

HEAT AND THERMODYNAMICS

Learning Objectives

At the end of this chapter the students will be able to:

1. State the basic postulates of Kinetic theory of gases.
2. Explain how molecular movement causes the pressure exerted by a gas and derive the equation $P = 2/3\ N_o < \frac{1}{2} mv^2 >$, where N_o is the number of molecules per unit volume of the gas.
3. Deduce that the average translational kinetic energy of molecules is proportional to temperature of the gas.
4. Derive gas laws on the basis of Kinetic theory.
5. Describe that the internal energy of an ideal gas is due to kinetic energy of its molecules.
6. Understand and use the terms work and heat in thermodynamics.
7. Differentiate between isothermal and adiabatic processes.
8. Explain the molar specific heats of a gas.
9. Apply first law of thermodynamics to derive $C_p - C_v = R$.
10. Explain the second law of thermodynamics and its meaning in terms of entropy.
11. Understand the concept of reversible and irreversible processes.
12. Define the term heat engine.
13. Understand and describe Carnot theorem.
14. Describe the thermodynamic scale of temperature.
15. Describe the working of petrol and diesel engines.
16. Explain the term entropy.
17. Explain that change in entropy $\Delta S = \pm \frac{\Delta Q}{T}$
18. Appreciate environmental crisis as an entropy crisis.

Thermodynamics deals with various phenomena of energy and related properties of matter, especially the transformation of heat into other forms of energy. An example of such transformation is the process converting heat into mechanical work. Thermodynamics thus plays central role in technology, since almost all the raw energy available for our use is liberated in the form of heat. In this chapter we shall study the behaviour of gases and laws of thermodynamics, their significance and applications.

11.1 KINETIC THEORY OF GASES

The behavior of gases is well accounted for by the kinetic theory based on microscopic approach. Evidence in favour of the theory is exhibited in diffusion of gases and Brownian motion of smoke particles etc.

The following postulates help to formulate a mathematical model of gases.

i. A finite volume of gas consists of very large number of molecules.

ii. The size of the molecules is much smaller than the separation between molecules.

iii. The gas molecules are in random motion and may change their direction of motion after every collision.

iv. Collision between gas molecules themselves and with walls of container are assumed to be perfectly elastic.

v. Molecules do not exert force on each other except during a collision.

Pressure of Gas

According to kinetic theory, the pressure exerted by a gas is merely the momentum transferred to the walls of the container per second per unit area due to the continuous collisions of molecules of the gas. An expression for the pressure exerted by a gas can, therefore, be obtained as follows:

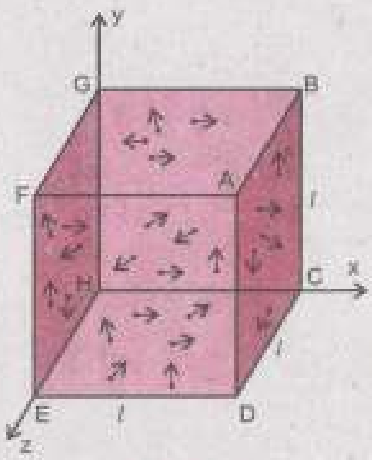


Fig. 11.1

Let a cubical vessel of side l, contains N molecules, each of mass m (Fig.11.1). The velocity **v**, of any one of these molecules can be resolved into three rectangular components v_{1x}, v_{1y}, v_{1z} parallel to three co-ordinate axes x, y and z.

Initial momentum of the molecule striking the face ABCDA is then mv_{1x}. If the collision is assumed perfectly elastic, the molecule will rebound from the face ABCDA with the same speed. Thus each collision produces a change in momentum, which is equal to

Final momentum - Initial momentum

or change in momentum $= -mv_{1x} - mv_{1x}$

$$\text{Change in momentum} = -2\,mv_{1x} \quad \text{.......... (11.1)}$$

After recoil the molecule travels to opposite face EFGHE and collides with it, rebounds and travels back to the face ABCDA after covering a distance $2l$. The time Δt between two successive collisions with face ABCDA is

$$\Delta t = \frac{2l}{v_{1x}} \quad \text{.......... (11.2)}$$

So the number of collisions per second that the molecule will make with this face is $= \frac{v_{1x}}{2l}$

Thus the rate of change of momentum of the molecule due to collisions with face ABCDA $= -2\,mv_{1x} \times \frac{v_{1x}}{2l} = \frac{-mv_{1x}^2}{l}$

The rate of change of momentum of the molecule is equal to the force applied by the wall. According to Newton's third law of motion, force F_{1x} exerted by the molecule on face ABCDA is equal but opposite, so

$$F_{1x} = \frac{-(-mv_{1x}^2)}{l} = \frac{mv_{1x}^2}{l}$$

Similarly the forces due to all other molecules can be determined. Thus the total x-directed force F_x due to N number of molecules of the gas moving with velocities v_1, v_2, v_3 ...... v_N is

$$F_x = F_{1x} + F_{2x} + F_{3x} + \ldots\ldots\ldots\ldots + F_{Nx}$$

or $$F_x = \frac{mv_{1x}^2}{l} + \frac{mv_{2x}^2}{l} + \frac{mv_{3x}^2}{l} + \ldots\ldots + \frac{mv_{Nx}^2}{l}$$

As pressure is normal force per unit area, hence pressure P_x on the face perpendicular to x-axis is

$$P_x = \frac{F_x}{A} = \frac{F_x}{l^2}$$

$$= \frac{1}{l^2}\left(\frac{mv_{1x}^2}{l} + \frac{mv_{2x}^2}{l} + \frac{mv_{3x}^2}{l} + \ldots\ldots\ldots + \frac{mv_{Nx}^2}{l}\right)$$

$$= \frac{m}{l^3}\left(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots + v_{Nx}^2\right) \ldots\ldots\ldots \quad (11.3)$$

As the mass of single molecule is m, the mass of N molecules will be mN.

Since density $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m\text{N}}{l^3}$

Hence, $$\frac{m}{l^3} = \frac{\rho}{\text{N}}$$

Substituting the value of $\frac{m}{l^3}$ in equation (11.3)

we get

$$P_x = \frac{\rho}{\text{N}}\left(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots + v_{Nx}^2\right)$$

or $$P_x = \rho\left(\frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots + v_{Nx}^2}{\text{N}}\right)$$

where $\left(\frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\ldots + v_{Nx}^2}{\text{N}}\right)$ is called the mean of squared velocities of the molecules moving along x direction, known as mean square velocity, represented by

$<v_x^2>$. Substituting $<v_x^2>$ in parenthesis of pressure expression

$$P_x = \rho < v_x^2 > \quad .... \quad (11.4)$$

Similarly pressure on the faces perpendicular to y and z axes will be $P_y = \rho < v_y^2 >$ and $P_z = \rho < v_z^2 >$

As there is no preference to one direction or another and molecules are supposed to be moving randomly, the mean square of all the component velocities will be equal. Hence

$$< v_x^2 > = < v_y^2 > = < v_z^2 >$$

and from vector addition $< v^2 > = < v_x^2 > + < v_y^2 > + < v_z^2 >$

thus, $$< v^2 > = 3 < v_x^2 >$$

or $$< v_x^2 > = \frac{1}{3} < v^2 >$$

putting this value of $< v_x^2 >$ in equation 11.4

$$P_x = \frac{\rho}{3} < v^2 >$$

We have considered the pressure on the face perpendicular to x-axis.

By Pascal's Law the pressure on the other sides and everywhere inside the vessel will be the same provided the gas is of uniform density. So

$$P_x = P_y = P_z = \frac{\rho}{3} < v^2 >$$

Thus in general

$$P = \frac{1}{3} \rho < v^2 >$$

Since density $\rho = \frac{mN}{V}$

Hence $$P = \frac{mN}{3V} <v^2>$$

or $$P = \frac{2}{3}\frac{N}{V} <\frac{1}{2}mv^2> \quad .... \quad (11.5)$$

$$P = \frac{2}{3}N_0 <\frac{1}{2}mv^2>$$

where N_0 is the number of molecules per unit volume.

Thus, $$P = \text{Constant} <K.E.>$$

or $$P \propto <K.E.>$$

While deriving the equation for pressure we have not accounted rotational and vibrational motion of molecules except the linear motion.

Hence pressure exerted by the gas is directly proportional to the average translational kinetic energy of the gas molecules.

Interpretation of Temperature

From experimental data the ideal gas law is deduced to be

$$PV = nRT \quad .... \quad (11.6)$$

Where n is the number of moles of the gas contained in volume V at absolute temperature T and R is called universal gas constant. Its value is 8.314 J mol^{-1} K^{-1}.

If N_A is the Avogadro number, then the above equation can be written as

$$PV = \frac{N}{N_A}RT$$

or $$PV = NkT \quad .... \quad (11.7)$$

where $k = R/N_A$ is the Boltzman constant. It is the gas constant per molecule and has the value = 1.38×10^{-23} J K^{-1}. Comparing equations 11.5 and 11.7

$$NkT = \frac{2}{3} N < \frac{1}{2} mv^2 >$$

or $$T = \frac{2}{3k} < \frac{1}{2} mv^2 > \quad \text{..........} \quad (11.8)$$

or $$T = \text{constant} < \text{K.E.} >$$

so $$T \propto < \text{K.E.} >$$

This relation shows that Absolute temperature of an ideal gas is directly proportional to the average translational kinetic energy of gas molecules.

We can, therefore, also say that average translational kinetic energy of the gas molecules shows itself macroscopically in the form of temperature.

Derivation of Gas Laws

(i) Boyle's Law

From kinetic theory of gases (Eq. 11.5)

$$PV = \frac{2}{3} N < \frac{1}{2} mv^2 >$$

If we keep the temperature constant, average K.E. i.e., $< 1/2\ mv^2 >$ remains constant, so the right hand side of the equation is constant.

Hence $$PV = \text{Constant}$$

or $$P \propto \frac{1}{V}$$

Thus pressure P is inversely proportional to volume V at constant temperature of the gas which is Boyle's law.

(ii) Charles' Law

Equation 11.5 can be written as

$$V = \frac{2}{3} \frac{N}{P} < \frac{1}{2} mv^2 >$$

If pressure is kept constant

$$V \propto \langle \frac{1}{2} mv^2 \rangle$$

As $$\langle \frac{1}{2} mv^2 \rangle \propto T$$

Hence $$V \propto T$$

Thus volume is directly proportional to absolute temperature of the gas provided pressure is kept constant. This is known as Charles' law.

Example 11.1: What is the average translational Kinetic energy of molecules in a gas at temperature 27C^o?

Solution:

Using Eq. 11.8 $$T = \frac{2}{3k} \langle K.E. \rangle$$

or $$\langle K.E. \rangle = \frac{3kT}{2}$$

where $$T = 27 + 273 = 300\ K$$
$$k = 1.38 \times 10^{-23}\ JK^{-1}$$

so $$\langle K.E. \rangle = \frac{3}{2} \times 1.38 \times 10^{-23}\ JK^{-1} \times 300\ K$$

$$= 6.21 \times 10^{-21}\ J$$

Example 11.2: Find the average speed of oxygen molecule in the air at S.T.P.

Solution: Under standard conditions

Temperature $$T = 0^0C = 273\ K$$

From Eq. 11.8

$$T = \frac{2}{3k} \langle \frac{1}{2} mv^2 \rangle$$

or $$\langle v^2 \rangle = \frac{3kT}{m}$$

Using Avogadro's number $N_A = 6.022 \times 10^{23}$, the mass m of one molecule of oxygen is

$$m = \frac{\text{molecular mass}}{N_A} = \frac{32\ g}{6.022 \times 10^{23}} = \frac{32\ kg}{6.022 \times 10^{26}}$$

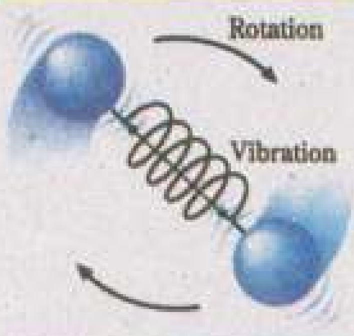
Substituting the values of k, T and m, we get

$$\langle v^2 \rangle = \frac{3 \times 1.38 \times 10^{-23}\ JK^{-1} \times 273\ K \times 6.022 \times 10^{26}}{32\ kg} = 212693\ m^2s^{-2}$$

or $$\langle v \rangle = 461\ ms^{-1}$$

11.2 INTERNAL ENERGY

Do You Know?



A diatomic gas molecule has both translational and rotational energy. It also has vibrational energy associated with the spring like bond between its atoms.

The sum of all forms of molecular energies (kinetic and potential) of a substance is termed as its internal energy. In the study of thermodynamics, usually ideal gas is considered as a working substance. The molecules of an ideal gas are mere mass points which exert no forces on one another. So the internal energy of an ideal gas system is generally the translational K.E. of its molecules. Since the temperature of a system is defined as the average K.E. of its molecules, thus for an ideal gas system, the internal energy is directly proportional to its temperature.

When we heat a substance, energy associated with its atoms or molecules is increased i.e., heat is converted to internal energy.

It is important to note that energy can be added to a system even though no heat transfer takes place. For example, when two objects are rubbed together, their internal energy increases because of mechanical work. The increase in temperature of the object is an indication of increase in the internal energy. Similarly, when an object slides over any surface and comes to rest because of frictional forces, the mechanical work done on or by the system is partially converted into internal energy.

In thermodynamics, internal energy is a function of state. Consequently, it does not depend on path but depends on initial and final states of the system. Consider a system which undergoes a pressure and volume change from P_a and V_a to P_b, and V_b respectively, regardless of the process by which the system changes from initial to final state. By experiment it has been seen that the change in internal energy is always the same and is independent of paths C_1 and C_2 as shown in the Fig. 11.2.

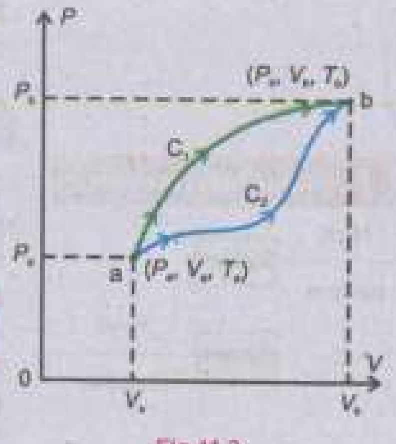


Fig.11.2

Thus internal energy is similar to the gravitational P.E. So like the potential energy, it is the change in internal energy and not its absolute value, which is important.

11.3 WORK AND HEAT

We know that both heat and work correspond to transfer of energy by some means. The idea was first applied to the steam engine where it was natural to pump heat in and get work out. Consequently it made a sense to define both heat in and work out as positive quantities. Hence work done by the system on its environment is considered +ive while work done on the system by the environment is taken as –ive. If an amount of heat Q enters the system it could manifest itself as either an increase in internal energy or as a resulting quantity of work performed by the system on the surrounding or both.

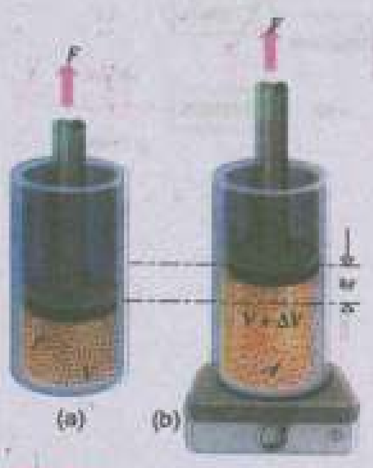


Fig. 11.3

A gas sealed in a cylinder by a weightless, frictionaless piston. The constant downward applied force F equals PA, and when the piston is displaced, downward work is done on the gas.

We can express the work in terms of directly measurable variables. Consider the gas enclosed in the cylinder with a moveable, frictionless piston of cross-sectional area A (Fig. 11.3 a). In equilibrium the system occupies volume V, and exerts a pressure P on the walls of the cylinder and its piston. The force F exerted by the gas on the piston is PA.

We assume that the gas expands through ΔV very slowly, so that it remains in equilibrium (Fig. 11.3 b). As the piston moves up through a small distance Δy, the work (W) done by the gas is

$$W = F\Delta y = PA\Delta y$$

Since $\qquad A\Delta y = \Delta V \qquad$ (Change in volume)

Hence $\qquad W = P\Delta V \qquad$ .... (11.9)

The work done can also be calculated by area of the curve under P-V graph as shown in Fig.11.4.

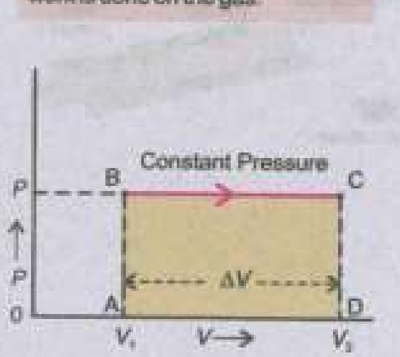
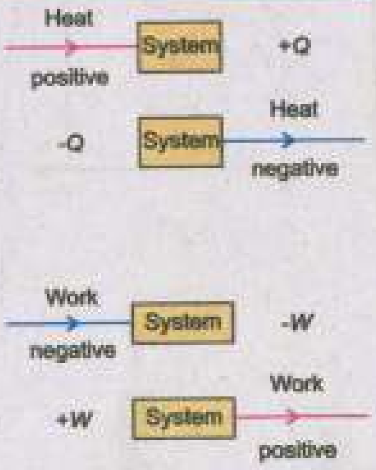


Fig.11.4

Knowing the details of the change in internal energy and the mechanical work done, we are in a position to describe the general principles which deal with heat energy and its transformation into mechanical energy. These principles are known as laws of thermodynamics.

11.4 FIRST LAW OF THERMODYNAMICS

For Your Information



When heat is added to a system there is an increase in the internal energy due to the rise in temperature, an increase in pressure or change in the state. If at the same time, a substance is allowed to do work on its environment by expansion, the heat Q required will be the heat necessary to change the internal energy of the substance from U_1 in the first state to U_2 in the second state plus the work W done on the environment.

Thus $$Q = (U_2 - U_1) + W$$

or $$Q = \Delta U + W \qquad \text{..........} \qquad (11.10)$$

Thus the change in internal energy $\Delta U = U_2 - U_1$ is defined as Q-W. Since it is the same for all processes concerning the state, the first law of thermodynamics, thus can be stated as,

In any thermodynamic process, when heat Q is added to a system, this energy appears as an increase in the internal energy ΔU stored in the system plus the work W done by the system on its surroundings.

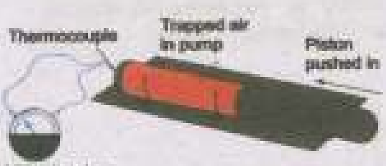


Fig. 11.5

A bicycle pump provides a good example. When we pump on the handle rapidly, it becomes hot due to mechanical work done on the gas, raising thereby its internal energy. One such simple arrangement is shown in Fig. 11.5. It consists of a bicycle pump with a blocked outlet. A thermocouple connected through the blocked outlet allows the air temperature to be monitored. When piston is rapidly pushed, thermometer shows a temperature rise due to increase of internal energy of the air. The push force does work on the air, thereby, increasing its internal energy, which is shown, by the increase in temperature of the air.

Human metabolism also provides an example of energy conservation. Human beings and other animals do work when they walk, run, or move heavy objects. Work requires energy. Energy is also needed for growth to make new cells and to replace old cells that have died. Energy transforming processes that occur within an organism are named as metabolism. We can apply the first law of thermodynamics,

$$\Delta U = Q - W$$

to an organism of the human body. Work (W) done will result in the decrease in internal energy of the body. Consequently the body temperature or in other words internal energy is maintained by the food we eat.

Example 11.3: A gas is enclosed in a container fitted with a piston of cross-sectional area 0.10 m^2. The pressure of the gas is maintained at 8000 Nm^{-2}. When heat is slowly transferred, the piston is pushed up through a distance of 4.0 cm. If 42 J heat is transferred to the system during the expansion, what is the change in internal energy of the system?

Solution:

The work done by the gas is

$$W = P\Delta V = PA\Delta y = 8000\ Nm^{-2} \times 0.10\ m^2 \times 4.0 \times 10^{-2}\ m$$

$$= 32\ Nm = 32\ J$$

The change in internal energy is found from first law of thermodynamics,

$$\Delta U = Q - W = 42\ J - 32\ J = 10\ J$$

Isothermal Process

It is a process which is carried out at constant temperature and hence the condition for the application of Boyle's Law on the gas is fulfilled. Therefore, when gas expands or compresses isothermally, the product of its pressure and volume during the process remains constant. If P_1, V_1 are initial pressure and volume where as P_2, V_2 are pressure and volume after the isothermal change takes place (Fig.11.6 a), then

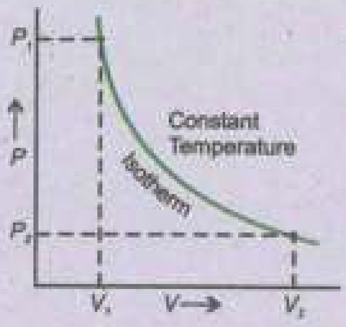


Fig.11.6(a)

$$P_1V_1 = P_2V_2$$

In case of an ideal gas, the P.E. associated with its molecules is zero, hence, the internal energy of an ideal gas depends only on its temperature, which in this case remains constant, therefore, $\Delta U = 0$. Hence, the first law of thermodynamics reduces to

$$Q = W$$

Thus if gas expands and does external work W, an amount of heat Q has to be supplied to the gas in order to produce an isothermal change. Since transfer of heat from one place to another requires time, hence, to keep the temperature of the gas constant, the expansion or compression must take place slowly. The curve representing an isothermal process is called an isotherm (Fig. 11.6a).

Adiabatic Process

An adiabatic process is the one in which no heat enters or leaves the system. Therefore, $Q = 0$ and the first law of thermodynamics gives

$$W = -\Delta U$$

Thus if the gas expands and does external work, it is done at the expense of the internal energy of its molecules and, hence, the temperature of the gas falls. Conversely an adiabatic compression causes the temperature of the gas to rise because of the work done on the gas.

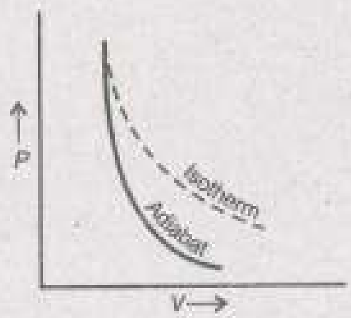


Fig. 11.6(b)

Adiabatic change occurs when the gas expands or is compressed rapidly, particularly when the gas is contained in an insulated cylinder. The examples of adiabatic processes are

(i) The rapid escape of air from a burst tyre.

(ii) The rapid expansion and compression of air through which a sound wave is passing.

(iii) Cloud formation in the atmosphere.

In case of adiabatic changes it has been seen that

$$PV^{\gamma} = \text{Constant}$$

where, γ is the ratio of the molar specific heat of the gas at constant pressure to molar specific heat at constant volume. The curve representing an adiabatic process is called an adiabat (Fig. 11.6 b).

11.5 MOLAR SPECIFIC HEATS OF A GAS

One kilogram of different substances contain different number of molecules. Sometimes it is preferred to consider a quantity called a mole, since one mole of any substance contains the same number of molecules. The molar specific heat of the substance is defined as the heat required to raise the temperature of one mole of the substance through 1 K. In case of solids and liquids the change of volume and hence work done against external pressure during a change of temperature is negligibly small. But same can not be said about gases which suffer variation in pressure as well as in volume with the rise in temperature. Hence, to study the effect of heating the gases, either pressure or volume is kept constant. Thus, it is customary to define the molar specific heats of a gas in two ways.

(i) The molar specific heat at constant volume is the amount of heat transfer required to raise the temperature of one mole of the gas through 1 K at constant volume and is symbolized by C_v.

If 1 mole of an ideal gas is heated at constant volume so that its temperature rises by ΔT, the heat transferred Q_v must be equal to $C_v\ \Delta T$. Because $\Delta V = 0$, no work is done (Fig 11.7 a). Applying first law of thermodynamics,

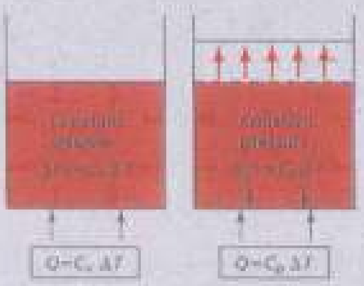


Fig. 11.7

$$Q_v = \Delta U + W$$

Hence, $\quad C_v\ \Delta T = \Delta U + 0$

or $$\Delta U = C_v\ \Delta T \quad \ldots\ldots\ldots \quad (11.11)$$

(ii) The molar specific heat at constant pressure is the amount of heat transfer required to raise the temperature of one mole of the gas through 1 K at constant pressure and it is represented by symbol C_p. To raise the temperature of 1 mole of the gas by ΔT at constant pressure, the heat transfer Q_p must be equal to $C_p\ \Delta T$ (Fig 11.7 b). Thus,

$$Q_p = C_p \,\Delta T \quad .... \quad (11.12)$$

Derivation of $C_p - C_v = R$

When one mole of a gas is heated at constant pressure, the internal energy increases by the same amount as at constant volume for the same rise in temperature ΔT. Thus from Eq. 11.11

$$\Delta U = C_v \,\Delta T$$

Since the gas expands to keep the pressure constant, so it does work $W = P\,\Delta V$, where ΔV is the increase in volume.

Substituting the values of heat transfer Q_p, internal energy ΔU and the work done W in Eq.11.10, we get

$$C_p \,\Delta T = C_v \,\Delta T + P\Delta V \quad .... \quad (11.13)$$

Using equation 11.6 for one mole of an ideal gas,

$$PV = RT \quad .... \quad (11.14)$$

At constant pressure P, amount of work done by one mole of a gas due to expansion ΔV (Fig. 11.7 b) caused by the rise in temperature ΔT is given by Eq. 11.14

$$P\,\Delta V = R\,\Delta T$$

Substituting for $P\,\Delta V$ in Eq. 11.13

$$C_p \,\Delta T = C_v \,\Delta T + R\,\Delta T$$

or $$C_p = C_v + R$$

or $$C_p - C_v = R \quad .... \quad (11.15)$$

It is obvious from Eq. 11.15 that $C_p > C_v$ by an amount equal to universal gas constant R.

11.6 REVERSIBLE AND IRREVERSIBLE PROCESSES

A reversible process is one which can be retraced in exactly reverse order, without producing any change in the surroundings. In the reverse process, the working substance passes through the same stages as in the direct process but thermal and mechanical effects at each stage are exactly reversed. If heat is absorbed in the direct process, it will be given out in the reverse process and if work is done by the substance in the direct process, work will be done on the substance in the reverse process. Hence, the working substance is restored to its original conditions.

A succession of events which bring the system back to its initial condition is called a cycle. A reversible cycle is the one in which all the changes are reversible.

Although no actual change is completely reversible but the processes of liquefaction and evaporation of a substance, performed slowly, are practically reversible. Similarly the slow compression of a gas in a cylinder is reversible process as the compression can be changed to expansion by slowly decreasing the pressure on the piston to reverse the operation.

If a process can not be retraced in the backward direction by reversing the controlling factors, it is an irreversible process.

All changes which occur suddenly or which involve friction or dissipation of energy through conduction, convection or radiation are irreversible. An example of highly irreversible process is an explosion.

11.7 HEAT ENGINE

A heat engine converts some thermal energy to mechanical work. Usually the heat comes from the burning of a fuel. The earliest heat engine was the steam engine. It was developed on the fact that when water is boiled in a vessel covered with a lid, the steam inside tries to push the lid off showing the ability to do work. This observation helped to develop a steam engine.

Do You Know?

The steam engine is a thermodynamics system.

Basically a heat engine (Fig. 11.8) consists of hot reservoir or source which can supply heat at high temperature and a cold reservoir or sink into which heat is rejected at a lower temperature. A working substance is needed which can absorb heat Q_1 from source, converts some of it into work W by its expansion and rejects the rest heat Q_2 to the cold reservoir or sink. A heat engine is made cyclic to provide a continuous supply of work.

11.8 SECOND LAW OF THERMODYNAMICS

Fig. 11.8

Schematic representation of a heat engine. The engine absorbs heat Q_1 from the hot reservoir, expels heat Q_2 to the cold reservoir and does work W.

First law of thermodynamics tells us that heat energy can be converted into equivalent amount of work, but it is silent about the conditions under which this conversion takes place. The second law is concerned with the circumstances in which heat can be converted into work and direction of flow of heat.

Before initiating the discussion on formal statement of the second law of thermodynamics, let us analyze briefly the factual operation of an engine. The engine or the system represented by the block diagram Fig. 11.8 absorbs a quantity of heat Q_1 from the heat source at temperature T_1. It does work W and expels heat Q_2 to low temperature reservoir at temperature T_2. As the working substance goes through a cyclic process, in which the substance eventually returns to its initial state, the change in internal energy is zero. Hence from the first law of thermodynamics, net work done should be equal to the net heat absorbed.

$$W = Q_1 - Q_2$$

In practice, the petrol engine of a motor car extracts heat from the burning fuel and converts a fraction of this energy to mechanical energy or work and expels the rest to atmosphere. It has been observed that petrol engines convert roughly 25% and diesel engines 35 to 40% available heat energy into work.

The second law of thermodynamics is a formal statement based on these observations. It can be stated in a number of different ways.

According to Lord Kelvin's statement based on the working of a heat engine

It is impossible to devise a process which may convert heat, extracted from a single reservoir, entirely into work without leaving any change in the working system.

This means that a single heat reservoir, no matter how much energy it contains, can not be made to perform any work. This is true for oceans and our atmosphere which contain a large amount of heat energy but can not be converted into useful mechanical work. As a consequence of second law of thermodynamics, two bodies at different temperatures are essential for the conversion of heat into work. Hence for the working of heat engine there must be a source of heat at a high temperature and a sink at low temperature to which heat may be expelled. The reason for our inability to utilize the heat contents of oceans and atmosphere is that there is no reservoir at a temperature lower than any one of the two.

Tidbits

According to the Kelvin statement of the second law of thermodynamics, the process pictured here is impossible. Heat from a source at a single temperature cannot be converted entirely into work.

11.9 CARNOT ENGINE AND CARNOT'S THEOREM

Sadi Carnot in 1840 described an ideal engine using only isothermal and adiabatic processes. He showed that a heat engine operating in an ideal reversible cycle between two heat reservoirs at different temperatures, would be the most efficient engine. A Carnot cycle using an ideal gas as the working substance is shown on *PV* diagram (Fig. 11.9). It consists of following four steps.

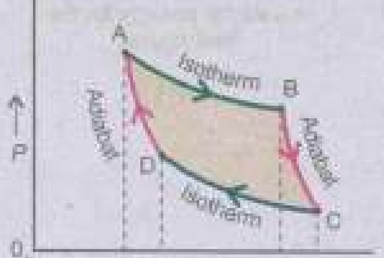


Fig.11.9

1. The gas is allowed to expand isothermally at temperature T_1, absorbing heat Q_1 from the hot reservoir. The process is represented by curve AB.
2. The gas is then allowed to expand adiabatically until its temperature drops to T_2. The process is represented by curve BC.
3. The gas at this stage is compressed isothermally at temperature T_2 rejecting heat Q_2 to the cold reservoir. The process is represented by curve CD.

4. Finally the gas is compressed adiabatically to restore its initial state at temperature T_1. The process is represented by curve DA.

Thermal and mechanical equilibrium is maintained all the time so that each process is perfectly reversible. As the working substance returns to the initial state, there is no change in its internal energy i.e. $\Delta U = 0$.

The net work done during one cycle equals to the area enclosed by the path ABCDA of the PV diagram. It can also be estimated from net heat Q absorbed in one cycle.

$$Q = Q_1 - Q_2$$

From 1st law of thermodynamics

$$Q = \Delta U + W$$

$$W = Q_1 - Q_2$$

The efficiency η of the heat engine is defined as

$$\eta = \frac{\text{Output (Work)}}{\text{Input (Energy)}}$$

thus, $$\eta = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{..........} \quad (11.16)$$

The energy transfer in an isothermal expansion or compression turns out to be proportional to Kelvin temperature. So Q_1 and Q_2 are proportional to Kelvin temperatures T_1 and T_2 respectively and hence,

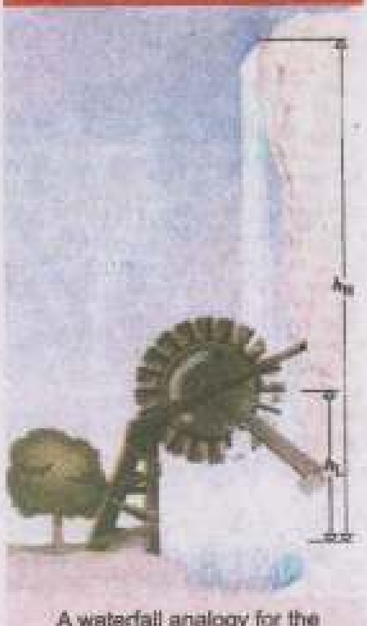
$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \quad \text{..........} \quad (11.17)$$

The efficiency is usually taken in percentage, in that case,

$$\text{percentage efficiency} = \left(1 - \frac{T_2}{T_1}\right)100$$

Thus the efficiency of Carnot engine depends on the temperature of hot and cold reservoirs. It is independent of the nature of working substance. The larger the

Interesting Information



A waterfall analogy for the heat engine.

temperature difference of two reservoirs, the greater is the efficiency. But it can never be one or 100% unless cold reservoir is at absolute zero temperature ($T_2 = 0$ K).

Such reservoirs are not available and hence the maximum efficiency is always less than one. Nevertheless the Carnot cycle establishes an upper limit on the efficiency of all heat engines. No practical heat engine can be perfectly reversible and also energy dissipation is inevitable. This fact is stated in Carnot's theorem

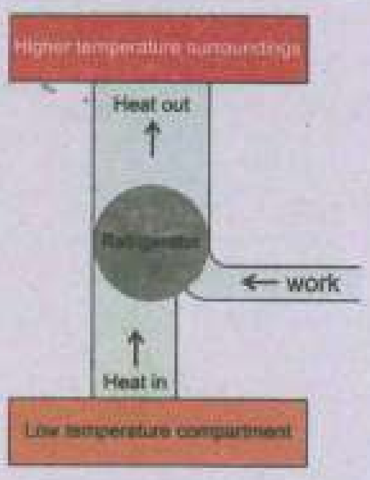
No heat engine can be more efficient than a Carnot engine operating between the same two temperatures.

The Carnot's theorem can be extended to state that,

All Carnot's engines operating between the same two temperatures have the same efficiency, irrespective of the nature of working substance.

In most practical cases, the cold reservoir is nearly at room temperature. So the efficiency can only be increased by raising the temperature of hot reservoir. All real heat engines are less efficient than Carnot engine due to friction and other heat losses.

Do You Know?



A refrigerator transfers heat from a low-temperature compartment to higher-temperature surroundings with the help of external work. It is a heat engine operating in reverse order.

Example 11.4: The turbine in a steam power plant takes steam from a boiler at 427°C and exhausts into a low temperature reservoir at 77°C. What is the maximum possible efficiency?

Solution:

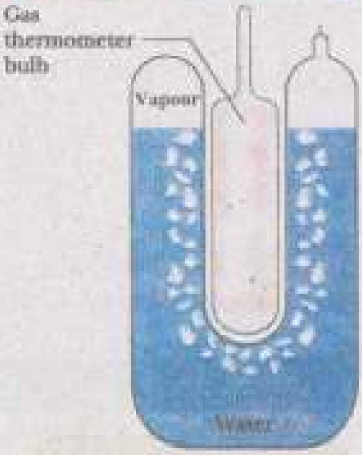
Maximum efficiency for any engine operating between temperatures T_1 and T_2 is

$$\eta = \frac{T_1 - T_2}{T_1}$$

where $T_1 = 427 + 273 = 700$ K

and $T_2 = 77 + 273 = 350$ K

For Your Information



A triple-point cell, in which solid ice, liquid water, and water vapour coexist in thermal equilibrium. By international agreement, the temperature of this mixture has been defined to be 273.16 K. The bulb of a constant-volume gas thermometer is shown inserted into the well of the cell.

so $$\eta = \frac{700\,K - 350\,K}{700\,K} = \frac{350\,K}{700\,K} = \frac{1}{2} = 0.5$$

or $$\eta = 50\%$$

11.10 THERMODYNAMIC SCALE OF TEMPERATURE

Generally a temperature scale is established by two fixed points using certain physical properties of a material which varies linearly with temperature. The Carnot cycle provides us the basis to define a temperature scale that is independent of material properties. According to it, the ratio Q_2/Q_1 depends only on the temperature of two heat reservoirs. The ratio of the two temperatures T_2/T_1 can be found by operating a reversible Carnot cycle between these two temperatures and carefully measuring the heat transfers Q_2 and Q_1. The thermodynamic scale of temperature is defined by choosing 273.16 K as the absolute temperature of the triple point of water as one fixed point and absolute zero, as the other. The unit of thermodynamic scale is kelvin. 1 K is defined as 1/273.16 of the thermodynamic temperature of the triple point of water. It is a state in which ice, water and vapour coexists in equilibrium and it occurs uniquely at one particular pressure and temperature. If heat Q is absorbed or rejected by the system at corresponding temperature T when the system is taken through a Carnot cycle and Q_3 is the heat absorbed or rejected by the system when it is at the temperature of triple point of water, then unknown temperature T in kelvin is given by

$$T = 273.16\,\frac{Q}{Q_3} \quad \text{..........} \quad (11.18)$$

Since this scale is independent of the property of the working substance, hence, can be applied at very low temperature.

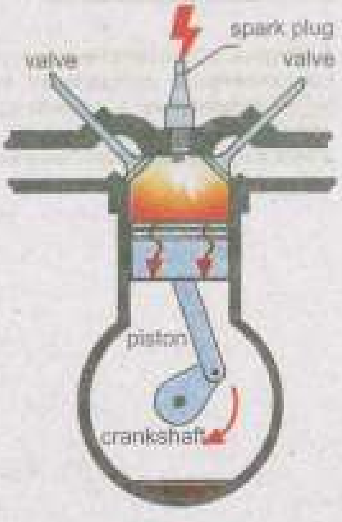


Fig. 11.10(a)

11.11 PETROL ENGINE

Although different engines may differ in their construction technology but they are based on the principle of a Carnot cycle. A typical four stroke petrol engine (Fig. 11.10 a) also undergoes four successive processes in each cycle.

1. The cycle starts on the intake stroke in which piston moves outward and petrol air mixture is drawn through an inlet valve into the cylinder from the carburetor at atmospheric pressure.
2. On the compression stroke, the inlet valve is closed and the mixture is compressed adiabatically by inward movement of the piston.
3. On the power stroke, a spark fires the mixture causing a rapid increase in pressure and temperature. The burning mixture expands adiabatically and forces the piston to move outward. This is the stroke which delivers power to crank shaft to drive the flywheels.
4. On the exhaust stroke, the outlet valves opens. The residual gases are expelled and piston moves inward.

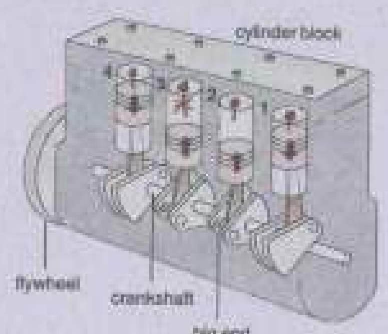


Fig. 11.10(b)

The cycle then begins again. Most motorbikes have one cylinder engine but cars usually have four cylinders on the same crankshaft (Fig 11.10 b). The cylinders are timed to fire turn by turn in succession for a smooth running of the car. The actual efficiency of properly tuned engine is usually not more than 25% to 30% because of friction and other heat losses.

Diesel Engine

No spark plug is needed in the diesel engine (Fig. 11.11). Diesel is sprayed into the cylinder at maximum compression. Because air is at very high temperature immediately after compression, the fuel mixture ignites on contact with the air in the cylinder and pushes the piston outward. The efficiency of diesel engine is about 35% to 40%.

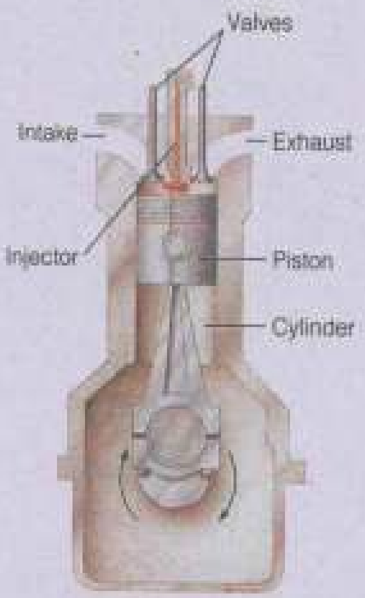


Fig. 11.11

11.12 ENTROPY

The concept of entropy was introduced into the study of thermodynamics by Rudolph Clausius in 1856 to give a quantitative basis for the second law. It provides another variable to describe the state of a system to go along with pressure, volume, temperature and internal energy. If a system undergoes a reversible process during which it absorbs a quantity of heat ΔQ at absolute temperature T, then the increase in the state variable called entropy S of the system is given by

$$\Delta S = \frac{\Delta Q}{T} \qquad .... \qquad (11.19)$$

Like potential energy or internal energy, it is the change in entropy of the system which is important.

Change in entropy is positive when heat is added and negative when heat is removed from the system. Suppose, an amount of heat Q flows from a reservoir at temperature T_1 through a conducting rod to a reservoir at temperature T_2 when $T_1 > T_2$. The change in entropy of the reservoir, at temperature T_1, which loses heat, decreases by Q/T_1 and of the reservoir at temperature T_2, which gains heat, increases by Q/T_2. As $T_1 > T_2$ so Q/T_2 will be greater than Q/T_1 i.e. $Q/T_2 > Q/T_1$.

Hence, net change in entropy = $\frac{Q}{T_2} - \frac{Q}{T_1}$ is positive.

It follows that in all natural processes where heat flows from one system to another, there is always a net increase in entropy. This is another statement of 2^{nd} law of thermodynamics. According to this law

If a system undergoes a natural process, it will go in the direction that causes the entropy of the system plus the environment to increase.

It is observed that a natural process tends to proceed towards a state of greater disorder. Thus, there is a relation between entropy and molecular disorder. For example an irreversible heat flow from a hot to a cold substance of a system increases disorder because the molecules are initially sorted out in hotter and cooler regions. This order is lost when the system comes to thermal equilibrium. Addition of heat to a system increases its disorder because of increase in average molecular speeds and therefore, the randomness of molecular motion. Similarly, free expansion of gas increases its disorder because the molecules have greater randomness of position after expansion than before. Thus in both examples, entropy is said to be increased.

We can conclude that only those processes are probable for which entropy of the system increases or remains constant. The process for which entropy remains constant is a reversible process; whereas for all irreversible processes, entropy of the system increases.

Every time entropy increases, the opportunity to convert some heat into work is lost. For example there is an increase in entropy when hot and cold waters are mixed. Then warm water which results cannot be separated into a hot layer and a cold layer. There has been no loss of energy but some of the energy is no longer available for conversion into work. Therefore, increase in entropy means degradation of energy from a higher level where more work can be extracted to a lower level at which less or no useful work can be done. The energy in a sense is degraded; going from more orderly form to less orderly form, eventually ending up as thermal energy.

In all real processes where heat transfer occurs, the energy available for doing useful work decreases. In other words the entropy increases. Even if the temperature of some system decreases, thereby decreasing the entropy, it is at the expense of net increase in entropy for some other system. When all the systems are taken together as the universe, the entropy of the universe always increases.

Example 11.5: Calculate the entropy change when 1.0 kg ice at 0°C melts into water at 0°C. Latent heat of fusion of ice $L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$.

Solution:

$$m = 1 \text{ kg}$$

$$T = 0°\text{C} = 273 \text{ K}$$

$$L_f = 3.36 \times 10^5 \text{ J kg}^{-1}$$

$$\Delta S = \frac{\Delta Q}{T}$$

where $\Delta Q = mL_f$

$$\Delta S = \frac{mL_f}{T}$$

$$\Delta S = \frac{1.00 \text{ kg} \times 3.36 \times 10^5 \text{ J kg}^{-1}}{273 \text{ K}}$$

$$\Delta S = 1.23 \times 10^3 \text{ J K}^{-1}$$

Thus entropy increases as it changes to water. The increase in entropy in this case is a measure of increase in the disorder of water molecules that change from solid to liquid state.

Do You Know?

Approximate efficiencies of various devices

Device	Efficiency (%)
Electric generator	70-99
Electric motor	50-93
Dry cell battery	90
Domestic gas furnace	70-85
Storage battery	72
Hydrogen-oxygen fuel cell	60
Liquid fuel rocket	47
Steam turbine	35-46
Fossil-fuel power plant	30-40
Nuclear power plant	30-35
Nuclear reactor	39
Aircraft gas turbine engine	36
Solid-state laser	30
Internal combustion gasoline engine	20-30
Gallium arsenide solar cells	>20
Fluorescent lamp	20
Silicon solar cell	12-16
Steam locomotive	8
Incandescent lamp	5
Watt's steam engine	1

11.13 ENVIRONMENTAL CRISIS AS ENTROPY CRISIS

The second law of thermodynamics provides us the key for both understanding our environmental crisis, and for understanding how we must deal with this crisis.

From a human standpoint the environmental crisis results from our attempts to order nature for our comforts and greed. From a physical standpoint, however, the environmental crisis is an entropy or disorder crisis resulting from our futile efforts to ignore the second law of thermodynamics. According to which, any increase in the order in a system will produce an even greater increase in entropy or disorder in the environment. An individual impact may not have a major consequence but an impact of large number of all individuals disorder producing activities can affect the overall life support system.

For Your Information

The jet engines on this aircraft convert thermal energy to work, but the visible exhaust clearly shows that a considerable amount of thermal energy is lost as waste heat.

The energy processes we use are not very efficient. As a result most of the energy is lost as heat to the environment. Although we can improve the efficiency but 2nd law eventually imposes an upper limit on efficiency improvement. Thermal pollution is an inevitable consequence of 2nd law of thermodynamics and the heat is the ultimate death of any form of energy. The increase in thermal pollution of the environment means increase in the entropy and that causes great concern. Even small temperature changes in the environment can have significant effects on metabolic rates in plants and animals. This can cause serious disruption of the overall ecological balance.

In addition to thermal pollution, the most energy transformation processes such as heat engines used for transportation and for power generation cause air pollution. In effect, all forms of energy production have some undesirable effects and in some cases all problems can not be anticipated in advance.

The imperative from thermodynamics is that whenever you do anything, be sure to take into account its present and possible future impact on your environment. This is an ecological imperative that we must consider now if we are to prevent a drastic degradation of life on our beautiful but fragile Earth.

SUMMARY

- From the Kinetic theory of gases $P = \frac{1}{3}\rho\langle v^2 \rangle$.
- The first law of thermodynamics states that energy is conserved.
- The sum of all forms of molecular energy present in a thermodynamic system is called its internal energy.
- Isothermal process is the process in which Boyle's law holds good.
- Adiabatic process is the one in which no thermal energy is added or extracted from the system.
- Molar specific heat at constant volume is the amount of heat required to raise the temperature of one mole of the gas through 1 K keeping volume constant.
- Molar specific heat at constant pressure is the amount of heat required to raise the temperature of one mole of the gas through 1 K keeping pressure constant.
- A heat engine is a device which converts a part of thermal energy into useful work.
- Efficiency of Carnot engine is $1 - \frac{T_2}{T_1}$.
- The second law of thermodynamics can be stated as
  - (i) There is no perpetual motion machine that can convert the given amount of heat completely into work.
  - (ii) The total entropy of any system plus that of its environment increases as a result of any natural process.
- Entropy change ΔS due to heat transfer ΔQ at absolute temperature T is given by
$$\Delta S = \pm \frac{\Delta Q}{T}.$$
- Thermal pollution is an inevitable consequence of 2nd law of thermodynamics.

QUESTIONS

11.1 Why is the average velocity of the molecules in a gas zero but the average of the square of velocities is not zero?

11.2 Why does the pressure of a gas in a car tyre increase when it is driven through some distance?

11.3 A system undergoes from state P_1V_1 to state P_2V_2 as shown in Fig 11.12. What will be the change in internal energy?

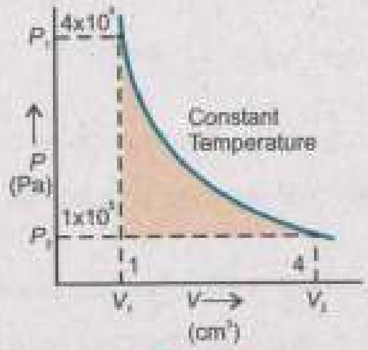


Fig.11.12

11.4 Variation of volume by pressure is given in Fig 11.13. A gas is taken along the paths ABCDA, ABCA and A to A. What will be the change in internal energy?

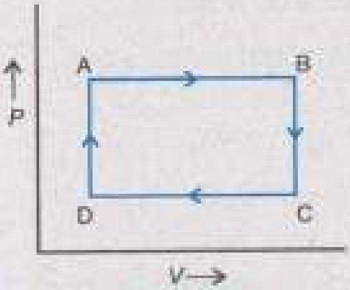


Fig.11.13(a)

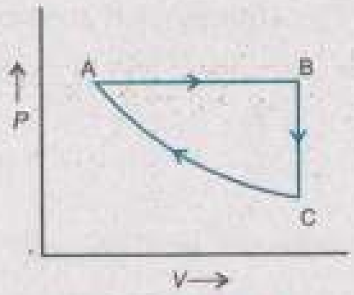


Fig.11.13(b)

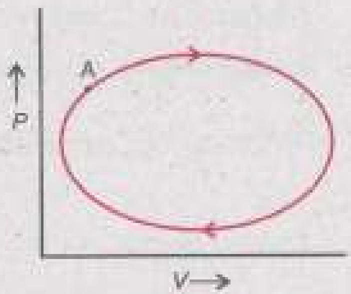


Fig.11.13(c)

11.5 Specific heat of a gas at constant pressure is greater than specific heat at constant volume. Why?

11.6 Give an example of a process in which no heat is transferred to or from the system but the temperature of the system changes.

11.7 Is it possible to convert internal energy into mechanical energy? Explain with an example.

11.8 Is it possible to construct a heat engine that will not expel heat into the atmosphere?

11.9 A thermos flask containing milk as a system is shaken rapidly. Does the temperature of milk rise?

11.10 What happens to the temperature of the room, when an airconditioner is left running on a table in the middle of the room?

11.11 Can the mechanical energy be converted completely into heat energy? If so give an example.

11.12 Does entropy of a system increase or decrease due to friction?

11.13 Give an example of a natural process that involves an increase in entropy.

11.14 An adiabatic change is the one in which

a. No heat is added to or taken out of a system

b. No change of temperature takes place

c. Boyle's law is applicable

d. Pressure and volume remains constant

11.15 Which one of the following process is irreversible?

a. Slow compressions of an elastic spring

b. Slow evaporation of a substance in an isolated vessel

c. Slow compression of a gas

d. A chemical explosion

11.16 An ideal reversible heat engine has

a. 100% efficiency

b. Highest efficiency

c. An efficiency which depends on the nature of working substance

d. None of these

NUMERICAL PROBLEMS

11.1 Estimate the average speed of nitrogen molecules in air under standard conditions of pressure and temperature.

(Ans: 493 ms^{-1})

11.2 Show that ratio of the root mean square speeds of molecules of two different gases at a certain temperature is equal to the square root of the inverse ratio of their masses.

11.3 A sample of gas is compressed to one half of its initial volume at constant pressure of 1.25×10^5 Nm^{-2}. During the compression, 100 J of work is done on the gas. Determine the final volume of the gas.

(Ans: 8×10^{-4} m^3)

11.4 A thermodynamic system undergoes a process in which its internal energy decreases by 300 J. If at the same time 120 J of work is done on the system, find the heat lost by the system.

(Ans: - 420 J)

11.5 A carnot engine utilises an ideal gas. The source temperature is 227°C and the sink temperature is 127°C. Find the efficiency of the engine. Also find the heat input from the source and heat rejected to the sink when 10000 J of work is done.

(Ans: 20%, 5.00×10^4 J, 4.00×10^4 J)

11.6 A reversible engine works between two temperatures whose difference is 100°C. If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.

(Ans: 100°C, 0°C)

11.7 A mechanical engineer develops an engine, working between 327°C and 27°C and claims to have an efficiency of 52%. Does he claim correctly? Explain.

(Ans: No)

11.8 A heat engine performs 100 J of work and at the same time rejects 400 J of heat energy to the cold reservoirs. What is the efficiency of the engine?

(Ans: 20%)

11.9 A Carnot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees the temperature of the source be increased?

(Ans: 373°C)

11.10 A steam engine has a boiler that operates at 450 K. The heat changes water to steam, which drives the piston. The exhaust temperature of the outside air is about 300 K. What is maximum efficiency of this steam engine?

(Ans: 33%)

11.11 336 J of energy is required to melt 1 g of ice at 0°C. What is the change in entropy of 30 g of water at 0°C as it is changed to ice at 0°C by a refrigerator?

(Ans: $-36.8\ J\ K^{-1}$)

Appendix 1

Standard Definitions of Base Units

Metre: The unit of length is named as metre. Before 1960 it was defined as the distance between two lines marked on the bar of an alloy of platinum (90%) and iridium (10%) kept under controlled conditions at the International Bureau of Weights and Measures in France. The 11^{th} General Conference on Weights and Measures (1960) redefined the standard metre as follows: One metre is a length equal to 1,650,763.73 wave lengths in vacuum of the orange red radiation emitted by the Krypton 86-atom. However, in 1983 the metre was redefined to be the distance traveled by light in vacuum during a time of 1/299,792,458 second. In effect, this latest definition establishes that the speed of light in vacuum is 299,792,458 ms^{-1}.

Kilogram: The unit of mass is known as kilogram. It is defined as the mass of a platinum (90%) and iridium (10%) alloy cylinder, 3.9 cm in diameter and 3.9 cm in height, kept at the International Bureau of Weights and Measures in France. This mass standard was established in 1901.

Second: The unit of time is termed as second. It is defined as 1/86400 part of an average day of the year 1900 A.D. The recent time standard is based on the spinning motion of electrons in atoms. This is since 1967 when the International Committee on Weights and Measures adopted a new definition of second, making one second equal to the duration in which the outer most electron of the cesium-133 atom makes 9,192,631,770 vibrations.

Kelvin: Temperature is regarded as a thermodynamic quantity, because its equality determines the thermal equilibrium between two systems. The unit of temperature is kelvin. It is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. It should be noted that the triple point of a substance means the temperature at which solid, liquid and vapour phases are in equilibrium. The triple point of water is taken as 273.16 K. This standard was adopted in 1967.

Ampere: The unit of electric current is ampere. It is that constant current which if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section and placed a metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. This unit was established in 1971.

Candela: The unit of luminous intensity is candela. It is defined as the luminous intensity in the perpendicular direction of a surface of 1/600000 square metre of a black body radiator at the solidification temperature of platinum under standard atmospheric pressure. This definition was adopted by the 13^{th} General Conference of Weights and measures in 1967.

Mole: The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12 (adopted in 1971). When this unit i.e. mole is used, the elementary entities must be specified; these may be atoms, molecules, ions, electrons, other particles or specified groups of such particles. One mole of any substance contains 6.0225×10^{23} entities.

Appendix 2

Possible Error in A Compound Quantity

(i) ERROR IN THE COMPOUND QUANTITY $z = x + y$

If the errors in the quantities x and y are Δx and Δy respectively, the possible sum is then;

$$x \pm \Delta x + y \pm \Delta y$$

The maximum possible error is when we have

$$x + \Delta x + y + \Delta y$$

or

$$x - \Delta x + y - \Delta y$$

Hence, the quantity can be expressed as $x + y \pm (\Delta x + \Delta y)$

i.e., the errors are added.

Hence, error in z = error in x + error in y (A 2.1)

(ii) ERROR IN THE COMPOUND QUANTITY $z = xy$

If the errors in the quantities x and y are Δx and Δy respectively, the compound quantity could be as large as $(x + \Delta x)(y + \Delta y)$ or as small as $(x - \Delta x)(y - \Delta y)$. The product is thus between about $xy + x\,\Delta y + y\,\Delta x + \Delta x\,\Delta y$ and $xy - x\,\Delta y - y\,\Delta x + \Delta x\,\Delta y$. If we neglect $\Delta x\,\Delta y$, as being small, then the error is between

$$x\,\Delta y + y\,\Delta x \quad \text{and} \quad -(x\,\Delta y + y\,\Delta x)$$

or

$$\pm (x\,\Delta y + y\,\Delta x)$$

The possible fractional error is thus

$$= \frac{\pm(x\Delta y + y\Delta x)}{xy} = \pm\left(\frac{\Delta y}{y} + \frac{\Delta x}{x}\right)$$

which is the sum of possible fractional errors. Since the fractional error is generally written as percentage error, hence the possible percentage error is the sum of the percentage errors for the product of the two physical quantities.

i.e., % error in z = % error in x + % error in y (A 2.2)

(iii) ERROR IN THE COMPOUND QUANTITY $z = k\,x^a\,y^b$

Let z, x and y be the numerical values of the physical quantities and k be a constant. Taking log of both sides;

$\log z = \log k + a \log x + b \log y$

Differentiating: $$\frac{dz}{z} = 0 + a\frac{dx}{x} + b\frac{dy}{y}$$

Multiply by 100

$$\left(\frac{dz}{z}\right)100 = a\left(\frac{dx}{x}\right)100 + b\left(\frac{dy}{y}\right)100$$

If dx, dy and dz represent the errors in the quantities x, y, and z respectively, then

$$\%\ \text{error in } z = a\ (\%\ \text{error in } x) + b\ (\%\ \text{error in } y) \qquad \text{(A 2.3)}$$

(iv) Error or Uncertainty from Graphs

To find uncertainty in an average value obtained by plotting graphs, the first step is to draw best straight line through the plotted points using a transparent ruler. The best straight line passes through as many of plotted points as possible or which leaves almost an equal distribution of points on either side of the line. The second step is to pivot a transparent ruler about the centre of best straight line to draw greatest and least possible slopes. If slope of best straight line is m and greatest and least slopes are m_1 and m_2 as illustrated in Fig. A 2.1, then evaluate m_1 - m and m_2 - m which ever of these is

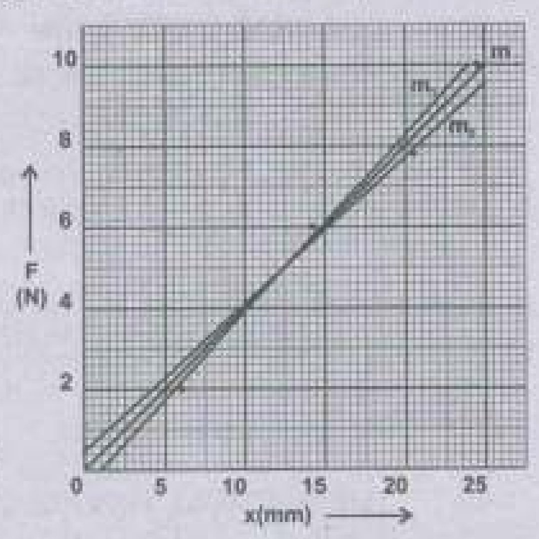


Fig. A 2.1

greater is the maximum possible uncertainty in the slope. If the intercept on a particular axis is required, the similar procedure can be followed.

Appendix 3

Mathematical Review

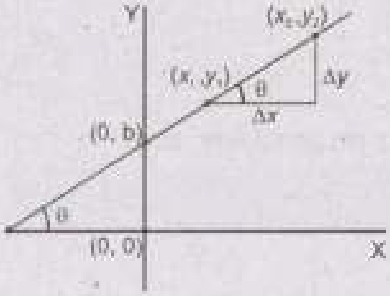


Fig. A 3.1

A. LINEAR EQUATION

A linear equation has the general form

$$y = ax + b \tag{A 3.1}$$

Where a and b are constants. This equation is referred to as being linear because the graph of y versus x is a straight line, as shown in Fig. A3.1. The constant b, called the intercept, represents the value of y at which the straight line intersects the Y-axis. The constant a is equal to the slope of the straight line and is also equal to the tangent of the angle that the line makes with the X-axis. If any two points on the straight line are specified by the coordinates (x_1, y_1) and (x_2, y_2), as in Fig. A 3.1, then the slope of the straight line can be expressed

$$\text{Slope} \quad a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \tan\theta \tag{A 3.2}$$

Note that a and b can be either positive or negative.

B. QUADRATIC EQUATION

The general form of a quadratic equation is

$$ax^2 + bx + c = 0 \tag{A 3.3}$$

where x is unknown quantity and a, b and c are numerical factors referred to as coefficients of the equation. This equation has two roots, given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{A 3.4}$$

If $b^2 > 4ac$, the roots will be real.

C. THE BINOMIAL THEOREM

(i) $$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2\times1}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3\times2\times1}a^{n-3}b^3 + \dots \tag{A 3.5}$$

(ii) If x < 1, then

$$(1 + x)^n = 1 + nx + \text{negligible terms} \qquad (A\ 3.6)$$

D. GEOMETRY

(i) Areas and volumes of some geometrical shapes are given in Table A3.1.

(ii) TABLE A 3.2

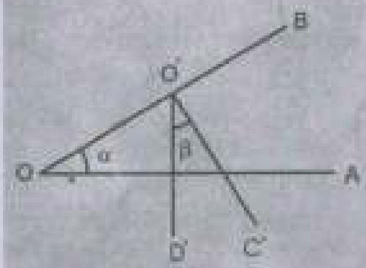
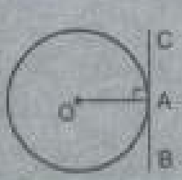
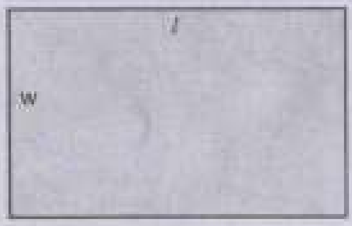
Condition	Theorem
	1. If AB is parallel to CD, then $\alpha = \beta$
	2. If O' C' is perpendicular to OB and O'D' is perpendicular to OA, then $\alpha = \beta$
	3. $\alpha + \beta + \gamma = 180^\circ$
	4. $\alpha = \beta + \gamma$
	5. The radius OA is perpendicular to the tangent line BC.

Table A 3.1

Areas and volumes of some geometrical shapes.

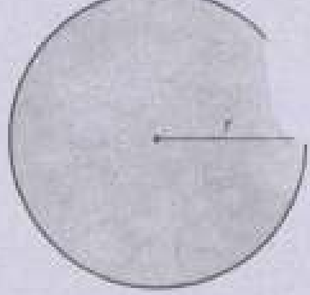


(i) Rectangle

Area = $l\,w$

(ii) Triangle

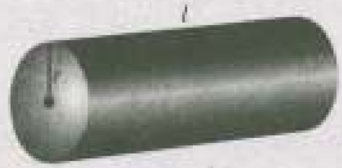
Area = $\frac{1}{2}bh$



(iii) circle

Area = πr^2

(Circumference = $2\pi r$)



(iv) Cylinder
volume = $\pi r^2 l$

(v) Rectangular box
volume = lwh

(vi) Sphere
Surface area = $4\pi r^2$

volume = $\frac{4}{3}\pi r^3$

E. TRIGONOMETRY

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{(A 3.7)}$$

$$\cos(90^\circ + \theta) = \sin\theta \quad \text{(A 3.8)}$$

$$\sin(90^\circ + \theta) = -\cos\theta \quad \text{(A 3.9)}$$

$$\sin(180^\circ - \theta) = \sin\theta \quad \text{(A 3.10)}$$

$$\cos(180^\circ - \theta) = -\cos\theta \quad \text{(A 3.11)}$$

$$\sin(\theta \pm \varphi) = \sin\theta\cos\varphi \pm \cos\theta\sin\varphi \quad \text{(A 3.12)}$$

$$\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi \quad \text{(A 3.13)}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \quad \text{(A 3.14)}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad \text{(A 3.15)}$$

According to our definitions, the trigonometric functions are limited to angles in the range [0, 90°]. We extend the meaning of these functions to negative or larger angles by a circle of unit radius, the unit circle (Fig.A 3.2). The angle is always measured with respect to the positive x axis counter clockwise positive and clockwise negative. The hypotenuse of the right angled triangle OAB is the radius of the unit circle. Its length is equal to 1, and it is always positive. The other two sides are assigned a sign according to the usual conventions i.e., positive to the right of the x-axis, and so on. With these conventions the trigonometric functions in each of the four quadrants have the signs listed in Table A 3.3.

If θ exceeds 360°, the whole pattern of signs and values repeats itself on the next pass around the circle. Thus, sine, cosine, and tangent are periodic functions of an angle with period 360°.

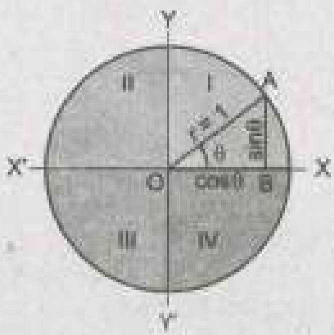


Fig. A 3.2

Table A 3.3

The Signs of the Trigonometric Functions

Quadrant	sinθ	cosθ	tanθ
I	+	+	+
II	+	-	-
II	-	-	+
IV	-	+	-

GLOSSARY

► **Adiabatic process**	A completely isolated process in which no heat transfer can take place.
► **Angular acceleration**	The rate of change of angular velocity with time.
► **Angular displacement**	Angle subtended at the centre of a circle by a particle moving along the circumference in a given time.
► **Angular momentum**	The cross product of position vector and linear momentum.
► **Angular velocity**	Angular displacement per second.
► **Antinode**	The point of maximum displacement on a stationary wave.
► **Artificial gravity**	The gravity like effect produced in orbiting space ship to overcome weightlessness.
► **Average acceleration**	Ratio of the change in velocity, that occurs within a time interval, to that time interval.
► **Average velocity**	Average rate at which displacement vector changes with time.
► **Base quantities**	Certain physical quantities such as length, mass and time.
► **Blue shift**	The shift of received wavelength from a star into the shorter region.
► **Bulk modulus**	Ratio of volumetric stress to volumetric strain.
► **Centre of mass**	The point at which all the mass of the body is assumed to be concentrated.
► **Centripetal force**	The force needed to move a body around a circular path.
► **Cladding**	A layer of lower refractive index (less density) over the central core of high refractive index (high density).
► **Compression**	The region of maximum density of a wave.
► **Conservative field**	The field in which work done along a closed path is zero.
► **Constructive interference**	When two waves meet each other in the same phase.
► **Core**	The central part of optical fibre which has relatively high refractive index (high density).
► **Crest**	The portion of a wave above the mean level.
► **Critical angle**	The angle of incidence for which the angle of refraction is 90°.
► **CRO**	A device used to display input signal into waveform.

- **Damping** — A process whereby energy is dissipated from the oscillatory system.
- **Denser medium** — The medium which has greater density.
- **Derived quantities** — The physical quantities defined in terms of base quantities.
- **Destructive interference** — When two waves overlap each other in opposite phases.
- **Diffraction** — Bending of light around obstacles.
- **Dimension** — One of the basic measurable physical property such as length, mass and time.
- **Displacement** — The change in the position of a body from its initial position to its final position.
- **Doppler shift** — The apparent change in the frequency due to relative motion of source and observer.
- **Drag force** — A retarding force experienced by an object moving through a fluid.
- **Elastic collision** — The interaction in which both momentum and kinetic energy conserve.
- **Energy** — Capacity to do work.
- **Entropy** — Measure of increase in disorder of a thermodynamic system or degradation of energy.
- **Escape velocity** — The initial velocity of a body to escape from Earth's gravitational field.
- **Forced oscillations** — The oscillations of a body subjected to an external force.
- **Free oscillations** — Oscillations of a body at its own frequency without the interference of an external force.
- **Freely falling body** — A body moving under the action of gravity only.
- **Fundamental mode** — Stationary wave setup with minimum frequency.
- **Geo-stationary satellite** — The satellite whose orbital motion is synchronized with the rotation of the Earth.
- **Harmonics** — Stationary waves setup with integral multiples of the fundamental frequency.
- **Heat engine** — A device that converts a part of input heat energy into mechanical work.
- **Ideal fluid** — An incompressible fluid having no viscosity.
- **Impulse** — The product of force and time for which it acts on a body.
- **Inelastic collision** — The interaction in which kinetic energy does not conserve.

- **Instantaneous acceleration** Acceleration at a particular instant of time.
- **Instantaneous velocity** Velocity at a particular instant of time.
- **Internal energy** The sum of all forms of molecular energies in a thermodynamic system.
- **Isothermal process** A process in which Boyle's law is applicable.
- **Kinetic energy** Energy possessed by a body due to its motion.
- **Laminar flow** Smooth sliding of layers of fluid past each other.
- **Least distance of distinct vision** The minimum distance from the eye at which an object can be seen distinctly.
- **Line spectrum** Set of discrete wavelengths.
- **Longitudinal wave** The wave in which the particles of the medium vibrate parallel to the propagation of the wave.
- **Magnification** The ratio of the angle subtended by the image as seen through the optical device to that subtended by the object at the unaided eye.
- **Modulus of elasticity** Ratio of stress and the strain.
- **Molar specific heat at constant pressure** Amount of heat needed to change the temperature of one mole of a gas through 1K keeping pressure constant.
- **Molar specific heat at constant volume** Amount of heat needed to change the temperature of one mole of a gas through 1K keeping volume constant.
- **Moment Arm** Perpendicular distance between the axis of rotation and line of action of the force.
- **Moment of inertia** The rotational analogue of mass in linear motion.
- **Momentum** The product of mass and velocity of an object.
- **Multi-mode graded index fibre** An optical fibre in which the central core has high refractive index which gradually decreases towards its periphery.
- **Node** The point of zero displacement.
- **Null vector** A vector of magnitude zero without any specific direction.
- **Orbital velocity** The tangential velocity to put a satellite in orbit around the Earth.
- **Oscillatory motion** To and fro motion of a body about its mean position.
- **Periodic motion** The motion which repeats itself after equal intervals of time.
- **Phase** A quantity which indicates the state and direction of motion of a vibrating particle.

► **Pitch**	The characteristics of sound by which a shrill sound can be distinguished from the grave sound.
► **Plane wavefront**	A disturbance lying in a plane surface.
► **Polarization**	The orientation of vibration along a particular direction.
► **Position vector**	A vector that describes the location of a point.
► **Potential energy**	Energy possessed by a body due to its position.
► **Power**	The rate of doing work.
► **Progressive wave**	The wave which transfers energy away from the source.
► **Projectile**	An object moving under the action of gravity and moving horizontally at the same time.
► **Radar speed trap**	An instrument used to detect the speed of moving object on the basis of Doppler shift.
► **Random error**	Error due to fluctuations in the measured quantity.
► **Range of a projectile**	The horizontal distance from the point where the projectile is launched to the point it returns to its launching height.
► **Rarefaction**	The region of minimum density.
► **Rarer medium**	The medium which has relatively less density.
► **Rays**	Radial lines leaving the point source in all directions.
► **Red shift**	The shift in the wavelength of light from a star towards longer wavelength region.
► **Resolving power**	The ability of an instrument to reveal the minor details of the object under examination.
► **Resonance**	A specific response of vibrating system to a periodic force acting with the natural period of the system.
► **Restoring force**	The force that brings the body back to its equilibrium position.
► **Resultant vector**	The sum vector of two or more vectors.
► **Root mean square velocity**	Square root of the average of the square of molecular velocities.
► **Rotational equilibrium**	A body having zero angular acceleration.
► **Scalar quantity**	A physical quantity that has magnitude only.
► **Scalar product**	The product of two vectors that results into a scalar quantity.
► **Significant figures**	The measured or calculated digits for a quantity which are reasonably reliable.

► **Simple harmonic motion**	A motion in which acceleration is directly proportional to displacement from mean position and is always directed towards the mean position.
► **Slinky spring**	A loose spring which has small initial length but a relatively large extended length.
► **Space time curvature**	Einstein's view of gravitation.
► **Spherical wavefront**	When the disturbance is propagated in all directions from a point source.
► **Stationary wave**	The resultant wave arising due to the interference of two identical but oppositely directed waves.
► **System international (SI)**	The internationally agreed system of units used almost world over.
► **Systematic error**	Error due to incorrect design or calibration of the measuring device.
► **Terminal velocity**	Maximum constant velocity of an object falling vertically downward.
► **Torque**	The turning effect of a force.
► **Total internal reflection**	When the angle of incidence increases by the critical angle, then the incident light is reflected back in the same material.
► **Trajectory**	The path through space followed by a projectile.
► **Translational equilibrium**	A body having zero linear acceleration.
► **Transverse wave**	The wave in which the particles of the medium vibrate perpendicular to the propagation of wave.
► **Trough**	The lower portion of a wave below the mean level.
► **Turbulent flow**	Disorderly and changing flow pattern of fluids.
► **Unit vector**	A vector of magnitude one used to denote direction.
► **Vector quantity**	A physical quantity that has both magnitude and direction.
► **Vector product**	The product of two vectors that results into another vector.
► **Wavefront**	A surface passing through all the points undergoing a similar disturbance (i.e., having the same phase) at a given instant.
► **Wavelength**	The distance between two consecutive wavefronts.
► **Work**	The product of magnitude of force and that of displacement in the direction of force.